

# MODELLING OF THE PROCEDURE FOR CHECKING THE FIXTURING STABILITY ACCOUNTING FOR WORKPIECE VIBRATORY BEHAVIOUR

Chaari, R.; AbdNadher, M.; Barkallah, M.; Louati, J. & Haddar, M.  
Mechanics Modelling and Production Research Unit (U2MP),  
Mechanical Engineering Department,  
National Engineering School of Sfax –TUNISIA.  
E-Mail: rchaari@yahoo.fr

## Abstract:

This paper presents a model-based framework for the verification of the dynamic stability of a fixtured workpiece during machining. The framework consists of a dynamic model for simulating the vibratory behaviour of the fixtured workpiece subjected to time- and space-varying machining loads. We present a static model for determining the localized fixture–workpiece contact deformations due to clamping, a geometric model for capturing the continuously changing geometry and inertia of the fixture–workpiece system during machining and a model for checking the dynamic stability of the fixtured workpiece.

A simulation example was solved to simulate the vibratory motion of the workpiece and to check the dynamic stability fixtured workpiece that is affected by the fixture–workpiece system dynamics and its continuous change during machining due to the material removal effect.

**Key Words:** Machining, Fixture design, Vibratory behaviour, Fixturing stability.

## 1. INTRODUCTION

Fixturing is an essential aspect of the manufacturing process. Fixture performance is crucial to product quality. A machining fixture has two basic functions: (1) to locate the component to a right position and orientation in relation to cutting tools; (2) to hold the component tightly so that it will not move during the machining. Therefore, the workpiece must remain stable in the fixture during machining in order to achieve operational safety and desired machining accuracy. In general, fixturing stability refers to the ability of the fixture to fully restrain the workpiece during the entire cutting process. As such, loss of contact and gross sliding during machining are considered to be indicators of an unstable workpiece and an inappropriate fixture design. These instabilities should be eliminated through correct design of the fixture. This paper presents a dynamic model that study the effects of fixture–workpiece system dynamics and the continuously change due to material removal on fixturing stability during machining. However, the majority of prior work treats the fixture–workpiece system as quasi-static and ignores the system dynamics. For example, Chou et al [1], DeMeter [2], Hurtado and Melkote [3], Tao et al. [4], Kang et al. [5], Liu and Strong [6].

In reality machining processes (e.g., milling) are often characterized by periodic forces. When the excitation frequency is near one of the natural frequencies (usually the lowest one) of the system, consideration of dynamic effects in fixturing analysis is crucial. Even if resonance does not occur during machining, a quasi-static analysis inadequate unless the excitation frequency is far below the system resonant frequency or large damping is present. Several researchers considered the dynamics of fixture–workpiece systems in their work. Mittal et al. [7] modeled a fixture–workpiece system using the Dynamic Analysis and Design System (DADS) software and modelled the fixture–workpiece contact as a lumped spring-damper-actuator element. However, they did not consider contact friction between the workpiece and fixture elements. Liao and Hu [8], [9] extended Mittal et al.'s work by including the workpiece compliance effect and contact friction through combined use of the finite

element (FE) method and DADS. In both studies, separation between the fixture elements and the workpiece during machining was checked for a given set of clamping forces, but macro-slip was not considered. In addition, neither of the papers presented a clamping force optimization model nor considered the effect of material removal. Hockenberger and DeMeter [10] developed meta-contact mechanics functions and applied them to simulate the dynamic behaviour of a fixture-workpiece system. Despite the aforementioned work that addressed fixturing dynamics, none considered the effect of material removal. Material loss and thus time-varying system inertia are characteristic of a machining process. Ignoring the chip removal effect can lead to erroneous predictions of machining fixturing stability especially when large-volume material removal is involved, e.g., machining of monolithic aerospace parts. Recently, Kaya and Öztürk [11] applied a FE-based element death technique to simulate the chip removal process for fixture layout verification. Liu and Strong [12] modeled the change of the workpiece gravity during machining. However, both papers treat the fixture-workpiece system as quasi-static.

This paper presents a model and procedure for verification of fixturing stability in machining. Unlike prior work, the effects of fixture-workpiece dynamics and material removal are considered. The model predicts the workpiece vibrational behaviour in the presence of the aforementioned effects when subjected to periodic machining loads. A simulation example is presented to illustrate the approach and the stability verification procedure. Then through this simulation example, we demonstrate the importance of the take in account of the effects of fixture-workpiece system dynamics and its continuous change due to material removal on fixturing stability.

## 2. PROBLEM FORMULATION AND APPROACH

This study focuses on structurally rigid workpieces (e.g. a solid block) surrounded by L locators and C clamps as shown schematically in Figure 1.

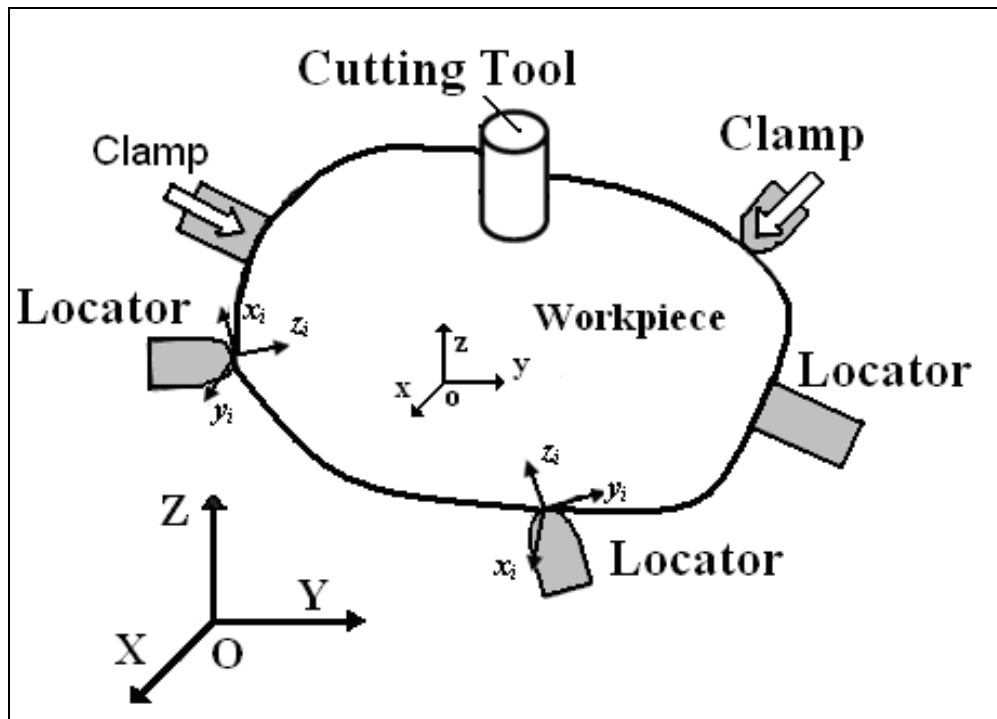


Figure 1: An arbitrary machining fixture-workpiece system [18].

Three coordinate systems are used to describe the position and orientation of the workpiece (see Fig.1): global coordinate system (XYZ), workpiece coordinate system (xyz), which is fixed to the mass center of the workpiece with its coordinate axes coinciding with the principal inertia axes of the initial workpiece shape, and local coordinate system ( $x_i y_i z_i$ ), which is fixed at each fixture–workpiece contact. Matrices are represented in the workpiece coordinate system (xyz) unless noted otherwise. The workpiece undergoes rigid body motion during clamping and machining due to elastic deformation, slip, and/or lift-off at the contact regions. From the standpoint of operational safety and machining accuracy, lift-off and macro-slip at the fixture–workpiece contacts during machining are undesirable because they result in loss of total restraint of the workpiece in the fixture. To eliminate these two types of fixturing instabilities, the dynamic stability of the fixtured workpiece during machining must be properly assured. This procedure consists of a dynamic model for simulating the vibratory behaviour of the fixtured workpiece subjected periodic machining loads and computing the dynamic displacements at the fixture–workpiece contacts due to machining, a geometric model for capturing the continuous change of the system geometry and inertia during machining due to the material removal effect and a static model for calculating the static deformation at the fixture–workpiece contacts due to clamping.

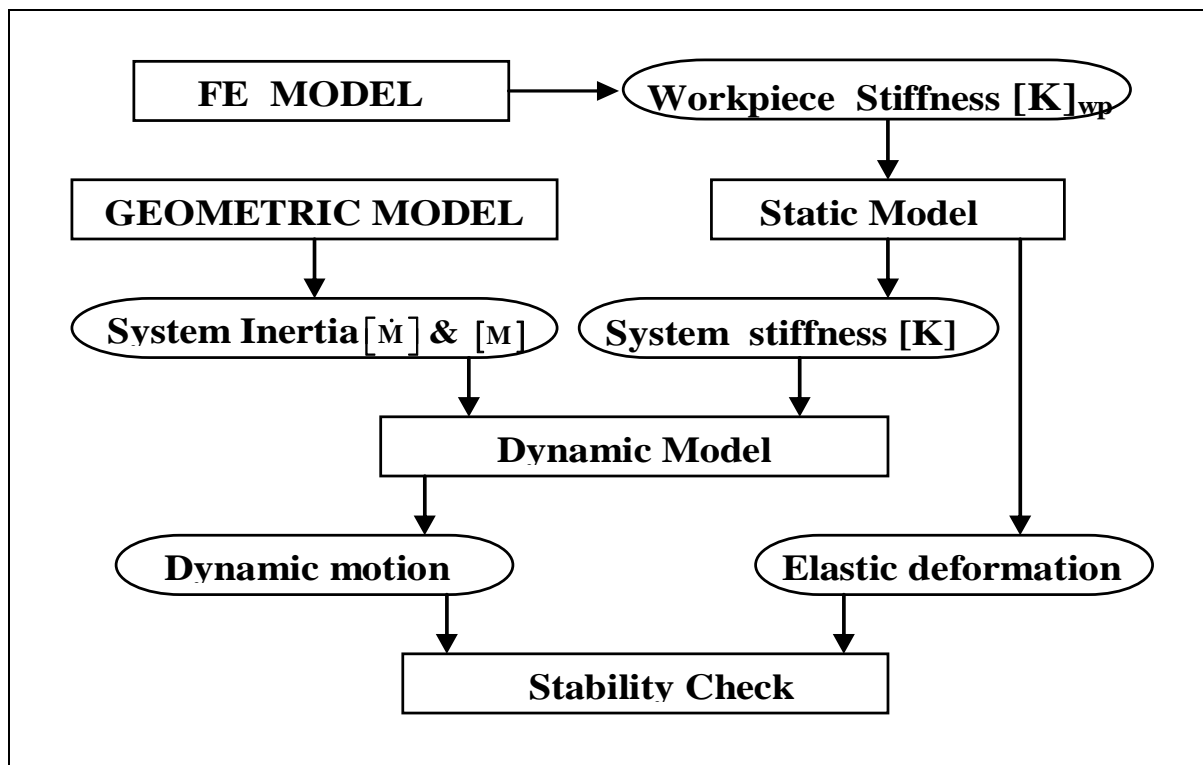


Figure 2: Procedure for analysis of fixturing dynamic in machining.

Therefore, the problem is to predict the stability of the workpiece characterized by its vibrational motion during machining given clamping and cutting loads. Specifically, during machining the status of each contact should be monitored to detect any undesired situations such as gross sliding or lift-off.

### 3. DYNAMIC MODEL OF THE FIXTURE WORKPIECE

#### 3.1 Dynamic model

A multi-tooth machining operation such as milling is characterized by periodic cutting forces whose frequency of variation equals the tooth passing frequency given by (1). Therefore, the fixture–workpiece system is subjected to forced vibration caused by the dynamic cutting forces.

$$f = z (N/60) \quad (1)$$

Where,  $z$  is the number of teeth in the cutting tool and  $N$  is the spindle speed in rpm.

When the excitation frequency is near the natural frequency of the fixture–workpiece system, the workpiece undergoes large motion that cannot be correctly predicted by a quasi-static analysis. Therefore, a dynamic model that is capable of predicting the vibrational behaviour of the fixture–workpiece system during cutting is required to analyze the fixturing stability in high-speed machining. Refer to [18] and [14], the model given in equation (2), was derived using the Lagrange's energy method.

$$[M] \cdot \ddot{\bar{q}} + [\dot{M}] \cdot \dot{\bar{q}} + [K] \cdot \bar{q} = \bar{Q}(t) \quad (2)$$

where,  $\bar{q} = [X \ Y \ Z \ \alpha \ \beta \ \gamma] \in R^6$  the workpiece rigid body motion vector;

$[M] = \begin{bmatrix} m(t) \cdot U_3 & 0 \\ 0 & I(t) \end{bmatrix} \in R^{6 \times 6}$  is the inertia matrix;  $[\dot{M}] = d[M]/dt$ ;  $[K]$  is the intrinsic

stiffness matrix of the fixture–workpiece system;  $\bar{Q}(t) = \begin{Bmatrix} \bar{F}(t) \\ \bar{I}(t) + \bar{r}_m(t) \times \bar{F}_t \end{Bmatrix} \in R^6$  is the vector of

external cutting loads;  $\bar{r}_m(t)$  is the distance between the instantaneous tool contact point with the machined surface and the center gravity of the workpiece.

Solving equation (2) yields  $\bar{q}(t)$  which is subsequently transformed into the contact dynamic displacements using equation (3).

$$\bar{d}(t) = [S]^T \bar{q}(t) \quad (3)$$

Where,  $\bar{d} = [\bar{d}_1^T, \bar{d}_2^T, \dots, \bar{d}_i^T, \dots, \bar{d}_{(L+c)}^T]^T \in R^{6 \times 3}$  with  $\bar{d}_i = [d_{xi} \ d_{yi} \ d_{zi}]^T \in R^3$ , and

$[S] = [[S_1], [S_2], \dots, [S_i], \dots, [S_{L+c}]] \in R^{6 \times 3(L+c)}$  with

$$[S_i] = \begin{bmatrix} \bar{S}_{xi} & \bar{S}_{yi} & \bar{S}_{zi} \\ \bar{\rho}_i \times \bar{S}_{xi} & \bar{\rho}_i \times \bar{S}_{yi} & \bar{\rho}_i \times \bar{S}_{zi} \end{bmatrix} \in R^{6 \times 3} \quad (4)$$

In equation 3,  $\bar{S}_{xi}$ ,  $\bar{S}_{yi}$  and  $\bar{S}_{zi}$  are the direction vectors of the  $x_i, y_i, z_i$  axes with respect to the (xyz) frame, and  $\bar{\rho}_i$  is the position vector of the  $i^{th}$  contact defined in (xyz). The matrix  $[S]$  is called the system configuration matrix and it depends only on the fixture layout and workpiece geometry.

#### 3.2 Geometric model

The effect of material removal on the fixture-workpiece system dynamics was considered and captured by a geometric model, which extracts the instantaneous system inertia ( $[M]$  and  $[\dot{M}]$ ) and geometry information as material is removed from the workpiece. The basic approach involves discretizing a tool pass into a series of increments. For each tool increment, the following steps are performed:

- Sweep the cutting tool along the tool path to generate a solid body representing the tool swept volume;
- Subtract the tool swept volume from the workpiece to obtain the machined workpiece;
- Analyze the machined workpiece to obtain its geometry and inertia information such as volume, center of gravity, surface normal, principal moments of inertia, and orientations of the principal inertia axes;
- Compute  $[M]$  and  $[\dot{M}]$  using the information obtained in the previous step.

#### 4. THE STATIC MODEL

By a developed static model we compute the static fixture–workpiece contact elastic deformation due to clamping, and we obtain the stiffness matrix of the fixture–workpiece system. Three virtual springs are used to model the contact stiffness with one in the normal ( $z_i$ ) direction and two in the tangential ( $x_i$  and  $y_i$ ) directions. The spring constants,  $k_{x_i}$ ,  $k_{y_i}$ , and  $k_{z_i}$ , are derived from force-displacement relationships reported in the contact mechanics literature. The effect of clamping forces on the contact stiffness is considered.

At each fixture-workpiece interface, the overall compliance comes from three sources: fixture- element, contact, and workpiece. Assuming that each source of compliance can be modeled as three linear springs in the  $x_i$ ,  $y_i$  and  $z_i$  directions of the local frame, respectively. The torsional compliance from all sources is considered to be negligible.

Therefore, as shown in Figure 3, the composite stiffness at the  $i^{\text{th}}$  fixture-workpiece contact,  $p_i$ , is the summation in series of three stiffness components,  $k_{ijf}$ ,  $k_{ijc}$  and  $k_{ijw}$  ( $j=x, y$  and  $z$ ), representing the stiffness of the fixture element, contact and workpiece, respectively.

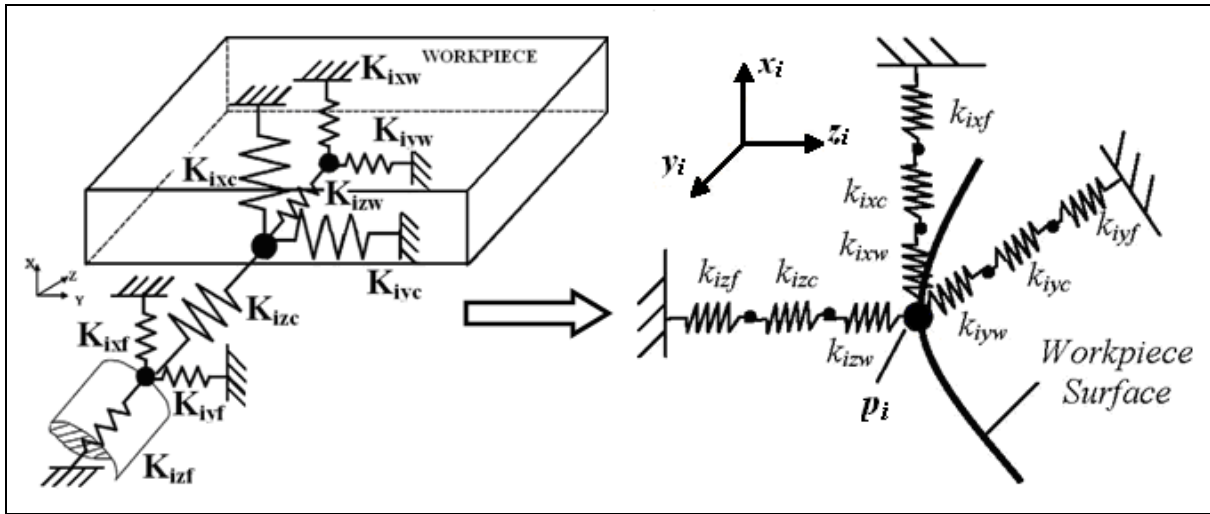


Figure 3: Composite stiffness at the  $i^{\text{th}}$  fixture-workpiece contact.

As shown in Figure 3 the workpiece structural compliance at the fixture-workpiece contact is approximated by three linear springs and is incorporated into the overall of the system by placing them in series with other springs representing the contact and fixture element compliances.

The calculation of the local stiffness of the fixture-workpiece system is given by this equation:

$$k_{ij} = \left[ (k_{ijf})^{-1} + (k_{ijc})^{-1} + (k_{ijw})^{-1} \right]^{-1} \quad (5)$$

The spring constants for the workpiece structural stiffness ( $k_{ixw}$ ,  $k_{iyw}$  and  $k_{izw}$  for  $i=1$  to 8) are obtained using a finite element (FE) method. The fixture-workpiece contact stiffness,  $k_{ijc}$  can be obtained by this equation (referring to [16]):

$$k_{zc} = \frac{3}{2} \left( \frac{16R(E^*)^2}{9} \right)^{1/3} P^{1/3} \quad (6)$$

$$k_{jc} = 8a \left( \frac{2-\nu_w}{G_w} + \frac{2-\nu_f}{G_f} \right)^{-1} \quad \text{for } j = x \text{ or } y$$

Where  $\frac{1}{E^*} = \frac{(1-\nu_w^2)}{E_w} + \frac{(1-\nu_f^2)}{E_f}$

$\nu$ ,  $E$  and  $G$  represent the Poisson's ratio, Young's modulus, and shear modulus of the material, respectively.

A fixture element (locator or clamp) is modeled as a short or long cylindrical cantilever, depending on its length-to-diameter ratio. The structural stiffness of a fixture element is calculated by the following expression (referring to [16]):

$$k_{ixf} = k_{iyf} = \begin{cases} (3E_{if}I_{if})/L_{if}^3, & (L_{if}/d_{if}) \geq 3 \\ (3G_{if}A_{if})/4L_{if}, & (L_{if}/d_{if}) < 3 \end{cases} \quad (7)$$

$$k_{izf} = \frac{E_{if}A_{if}}{I_{if}}$$

Where  $E_{if}$ ,  $G_{if}$ ,  $L_{if}$ ,  $A_{if}$  and  $I_{if}$  are the Young's modulus, shear modulus, length, cross-sectional area, diameter and polar moment of inertia of the  $i^{\text{th}}$  fixture element, respectively.

With  $k_{xi}$ ,  $k_{yi}$  and  $k_{zi}$  known, the stiffness matrix of the fixture-workpiece system can be computed as follows:

$$[K] = [S][K_c][S]^T \in R^{6 \times 6} \quad (8)$$

where  $[K_c] = \text{Blockdiag} \left[ \text{Diag}(k_{x1}, k_{y1}, k_{z1}), \dots, \text{Diag}(k_{xi}, k_{yi}, k_{zi}), \dots \right] \in R^{3(L+c) \times 3(L+c)}$

The principle of minimum complementary energy [14, 15] is utilized to develop the static model so that it can deal with an arbitrarily configured fixture-workpiece system without suffering from static indeterminacy of problems. The static model is then essentially a constrained nonlinear optimization model. The objective function of the model is the total complementary energy of the fixture-workpiece system subjected to clamping forces, and the design variables are the unknown contact forces of the system. Since the structural compliance of the workpiece and fixture elements are considered to be negligible relative to the contact compliance, the total complementary energy of the system is given by the sum of the stress energy of the compressed nonlinear virtual springs used to model the contact stiffness. Note that only constant clamping forces generated by force-controlled fixture are considered in this study. Therefore, denoting  $P_i$ ,  $Q_{xi}$ , and  $Q_{yi}$  as the contact forces due to clamping loads in the  $z_i$ ,  $x_i$ , and  $y_i$  directions, the static model can be written as:

minimise  $\Pi_c$

$(P_i, Q_{xi}, Q_{yi})$

$$= \sum_{i=1}^{(L+c)} \left( \int_0^{P_i} \delta z_i dP_i + \int_0^{Q_{xi}} \delta x_i dQ_{xi} + \int_0^{Q_{yi}} \delta y_i dQ_{yi} \right) \quad (9)$$

subject to :

$$\sum \bar{F} = \bar{0}, \quad \sum \bar{M} = \bar{0} \quad (10)$$

$$P_j = F_{cj} \text{ for } j=(L+1), \dots, (L+C). \quad (11)$$

$$\sqrt{(Q_{xi})^2 + (Q_{yi})^2} - \mu_s^i P_i \leq 0 \quad \text{for } i=1, \dots, (L+c) \quad (12)$$

$$P_i > 0 \quad \text{for } i=1, \dots, (L+c) \quad (13)$$

$$P_i - S_y (\Pi a_i^2) \leq 0 \quad \text{for } i=1, \dots, (L+c) \quad (14)$$

The constraint (10) represents the static equilibrium condition; (11) results from the fact that constant clamping forces are used; (12) represents the Coulomb friction law; (13) comes from the unilateral nature of a fixture–workpiece contact; (14) represents the non-yielding constraint on the contact stress where  $S_y$  is the yield strength of the workpiece material while  $a_i$  is the radius of the  $i^{\text{th}}$  contact region and is a function of the normal force  $P_i$ . The forces are then substituted into the contact force-displacement relationship to calculate the contact elastic deformation due to clamping. The contact force-displacement relationship for a spherical-tipped fixture element pressed against a curved workpiece surface is given as follows [16]:

$$\delta_z = \left[ \frac{9P^2}{16R(E^*)^2} \right], \quad \delta_j = \frac{Q_j}{8a} \left( \frac{2-\nu_w}{G_w} + \frac{2-\nu_f}{G_f} \right) \quad (15)$$

For  $j=x, y$

where,  $R = \left( \frac{1}{R_w} + \frac{1}{R_f} \right)^{-1}$  is the relative curvature at the contact with  $R_w$  being the local radius of

the workpiece surface and  $R_f$  being the tip radius of the fixture element;  $\frac{1}{E^*} = \frac{(1-\nu_w^2)}{E_w} + \frac{(1-\nu_f^2)}{E_f}$

,  $\nu$ ,  $E$  and  $G$  represent the Poisson's ratio, Young's modulus, and shear modulus of the material, respectively; the subscripts  $w$  and  $f$  refer to the workpiece and fixture elements, respectively;  $a = \left( \frac{3PR}{4E^*} \right)$  is the radius of the contact region.

## 5. FIXTURING STABILITY CRITERIA

The dynamic stability of the fixtured workpiece during machining can be analyzed by determining the status of the interaction between the workpiece and fixture elements (i.e. locators and clamps) at each contact. Prior to machining, the clamps are actuated and this causes localized elastic deformation at the fixture–workpiece contacts. Consequently, the workpiece assumes a static equilibrium position, from which it undergoes dynamic motion during machining. Superposition of the static elastic deformation due to clamping and the dynamic displacement due to machining at a fixture–workpiece contact gives the total motion of that contact. As shown in Fig.4, three types of contact status are possible: full stick, macro-slip and lift-off. Since total restraint of the workpiece by the fixture must be satisfied throughout the machining operation, lift-off of the workpiece from any fixture element and macro-slip of the workpiece at any contact at any instant are indicators of an unstable workpiece. In the local coordinate system ( $x_i, y_i, z_i$ ) for the  $i^{\text{th}}$  fixture–workpiece contact, lift-off is equivalent to a positive displacement of the workpiece in the  $z_i$  direction while macro-slip indicates that the Coulomb friction law is violated. Therefore, the two fixturing stability criteria can be stated mathematically as in equation (16), which must be satisfied for  $i=1$  to  $(L+C)$  during machining.

$$\begin{aligned} \max \{ \Delta_{z_i}(t) \} &\leq 0 \\ \max \left\{ \sqrt{[k_{x_i} \Delta_{x_i}(t)]^2 + [k_{y_i} \Delta_{y_i}(t)]^2} - \mu_s^i [k_{z_i} |\Delta_{z_i}(t)|] \right\} &\leq 0 \end{aligned} \quad (16)$$

where,  $t$  is the machining time;  $\Delta x_i(t)$ ,  $\Delta y_i(t)$  and  $\Delta z_i(t)$  are the superposed displacements of the  $i^{\text{th}}$  fixture–workpiece contact in the  $x_i$ ,  $y_i$ , and  $z_i$  directions, respectively;  $\mu_s^i$  is the static friction coefficient at the  $i^{\text{th}}$  contact;  $k_{x_i}$ ,  $k_{y_i}$ , and  $k_{z_i}$  are the local contact stiffnesses in the  $x_i$ ,  $y_i$ , and  $z_i$  directions, respectively

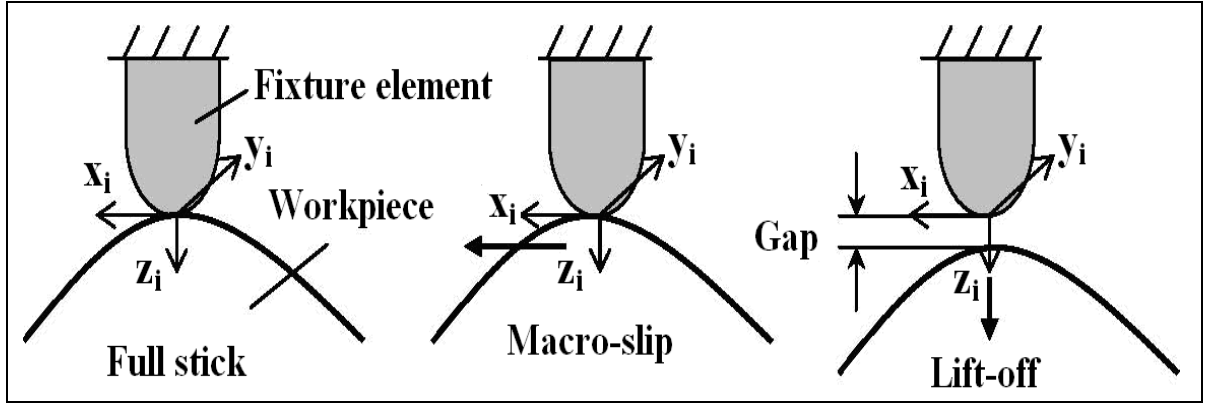


Figure 4: Dynamic contact interaction between workpiece and fixture element.

The total motion of the  $i^{\text{th}}$  is obtained as follows:

$$\Delta_{ij}(t) = d_{ij}(t) - \delta_{ij} \quad \text{For } j=x,y,z \quad (17)$$

Where, symbols  $d$  and  $\delta$  represent the dynamic and static displacements of the workpiece, respectively.

## 6. SIMULATION EXAMPLE

A flat end milling simulation example is used to illustrate the approach developed in this paper for to verify the dynamic stability of a fixtured workpiece during machining.

This example considers an end milling operation. As illustrated in Figure 5, the original workpiece is a solid block of aluminum 7075 and the operation involves milling a step cut on the top surface of the workpiece. The operation consists of 30 depth levels along the Z axis, with 20 tool passes along the Y axis at each level. Therefore, there are a total of 600 passes with the first pass at the right end of the workpiece. The removed volume is about 43% of the total volume of the original workpiece.

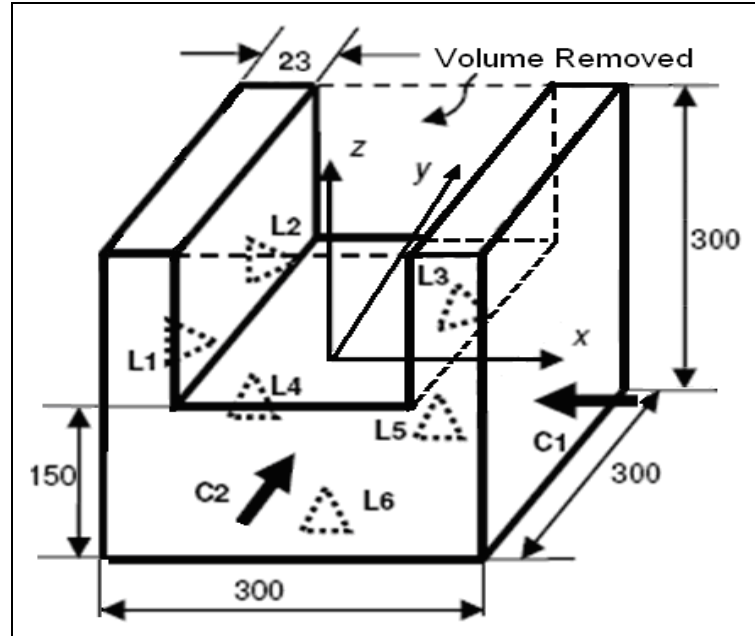


Figure 5: Final part and fixture layout (L1-L6: locators; C1-C2: clamps)  
(Note: all dimensions are in mm.).



Cutting conditions used in this example are given by the following Table I:

TableI: cutting conditions.

Feed Rate (mm/s)	Axial Depth (mm)	Radial Depth (mm)	Spindle Speed (rpm)
100	5	12.7	2500

(Tool: end mill, Ø 25.4 mm, 4-flute, and 30° helix)

The spatial coordinates of the fixture-workpiece contacts in the (xyz) frame are listed in Table II. All fixture elements are identical with a cylindrical body (radius=20 mm and length=30 mm) and a spherical tip (radius=19.8 mm).

Table II: Coordinates of fixture-workpiece contacts.

Locator	Coordinate(x, y, z)(mm)	Clamp	Coordinate(x, y, z)(mm)
L1	(-150, 50, -50)	C1	(150, 0, -50)
L2	(-150, -50, -50)	C2	(0, 150, -50)
L3	(0, -150, -50)		
L4	(-75, -75, -150)		
L5	(75, -75, -150)		
L6	(0, 75, -150)		

The material properties of the workpiece and the fixture elements are given in Table III.

Table III: Material properties.

Parameter	Workpiece	Fixture Elements
Material	Aluminun 7075-T6	steel 1018
Density(kg/m <sup>3</sup> )	2700	
Young's modulus (GPa)	70.3	201
Poisson's ratio	0.354	0.296
Yield strength (MP)	500	
Static coefficient of friction	0.35	

The instantaneous machining forces, shown in Figure 6 (during two tool revolutions), are obtained from an ideal milling force model derived from [17]. Note that the effect of the helix angle on the cutting forces is neglected in the force model, resulting in a zero force in the Z direction. In general, the cutting force in the Z (axial) direction in milling is small especially for the small axial depth of cut (5 mm) and relatively small helix angle (30°) employed in the current example.

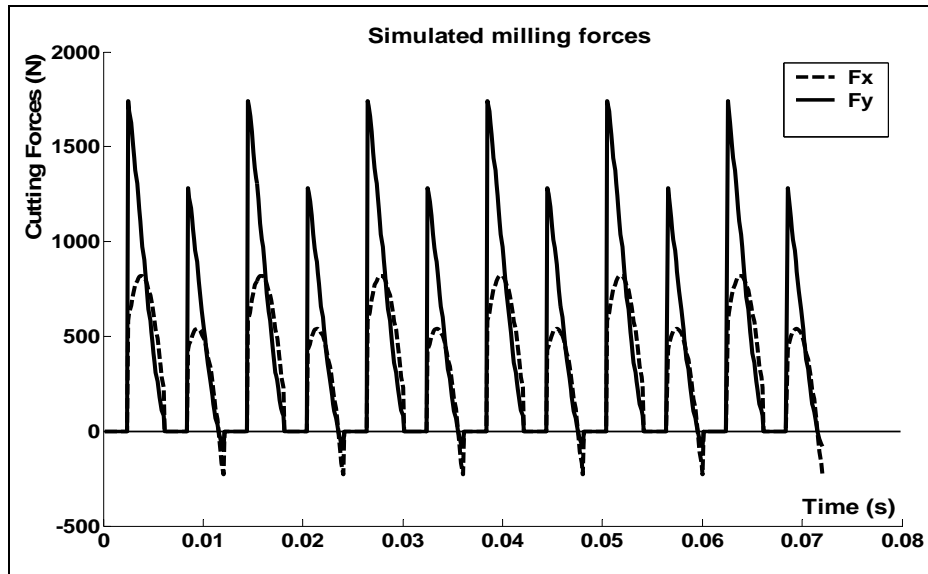


Figure 6: Simulated milling forces.

Results of the harmonic analysis of the fixture-workpiece system in the beginning and at the end of the milling operation are shown in Figure 7 as plots of the vibration amplitude vs. spindle speed. The three resonance regions in either plot in Figure 7 correspond to the three groups of natural frequencies of the fixture-workpiece system. It is seen that the resonance regions shift to the right as the workpiece loses material because the system natural frequencies become higher (see Table IV).

Table IV: Natural frequencies.

Natural Frequency (Hz)				
	1	2	3	4
Begening	213.7	244.6	284.8	348.76
End	281	321.7	374.5	458.7

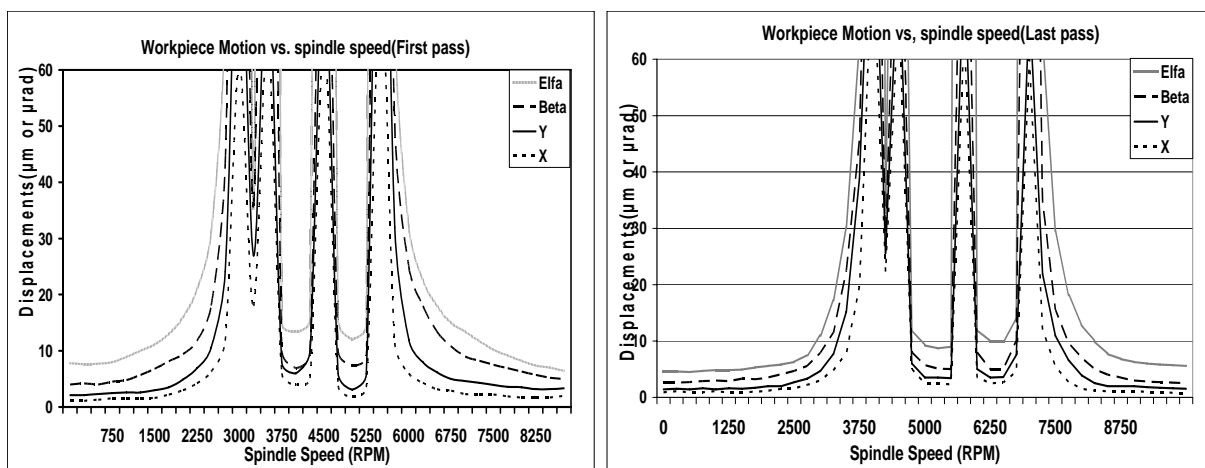


Figure 7: Workpiece motion versus spindle speed.

A spindle speed of 2500 rpm, which is in the curved region of the plots in Fig.7, is selected in this example so as to disqualify a quasi-static analysis but not in the immediate vicinity of the peak areas to avoid resonance.

As an example, the fixturing dynamic stability of the system operating under clamping forces of 1646 N (applied by C1) and 6082 N (applied by C2) during the first pass is shown in Figure 8. The horizontal axis of both plots in Figure 8 represents the fixture-workpiece contact indices, which range from 1 to 8 corresponding to L1, L2, L3, L4, L5, L6, C1, and C2 (Figure 5), respectively. The vertical axes of plots (a) and (b) in Figure 8 stand for the left hand sides of the two fixturing dynamic stability criteria (given in equation 16), respectively. Therefore, a stem above the zero horizontal line in Figure 8 (a) or (b) is an indicator of lift-off or macro-slip at the corresponding fixture-workpiece contact. The height of a stem represents the degree of fixturing dynamic stability (if below zero) or instability (if above zero) of the contact. It is seen from Fig.8 that the two clamping forces are unable to stabilize the workpiece during machining. Specifically, the workpiece lifts off at L6 and macro-slip occurs at L1, L2, and C1. With the help of Figure 5 and Figure 6, it can be concluded that the fixturing instabilities are due to the inappropriate combination of the two clamping forces. The clamping force applied by C2 (6082 N), pointing in the +Y direction, is much higher than that applied by C1 (1646 N) while the cutting force in the +Y direction is significantly higher than the cutting forces in the other two directions. Therefore, the workpiece is pushed hard against L3, resulting in the previously identified fixturing instabilities.

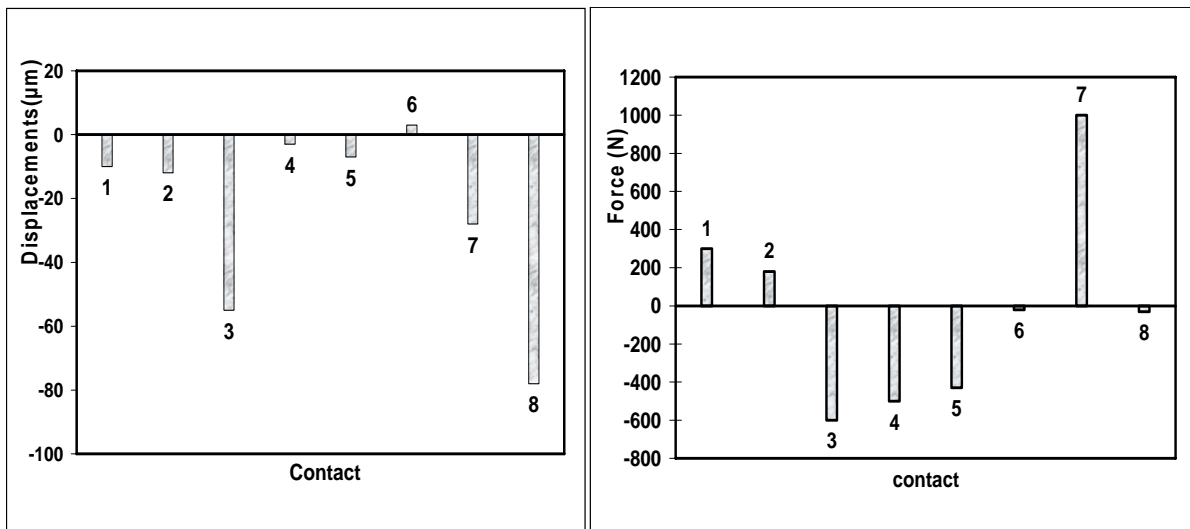


Figure 8: Fixturing dyanmic stability in the first pass under clamping forces 1646N and 6082N.

The dynamic motions of the workpiece during the first and last tool passes are presented on Figure 9 and Figure 10 (with the take account of the minimum clamped forces). Note that workpiece motions during only two tool revolutions are shown in each plot. As seen from these plots, the workpiece vibrations (including three translations and three rotations) change significantly during the last pass in which 43% material has been removed.

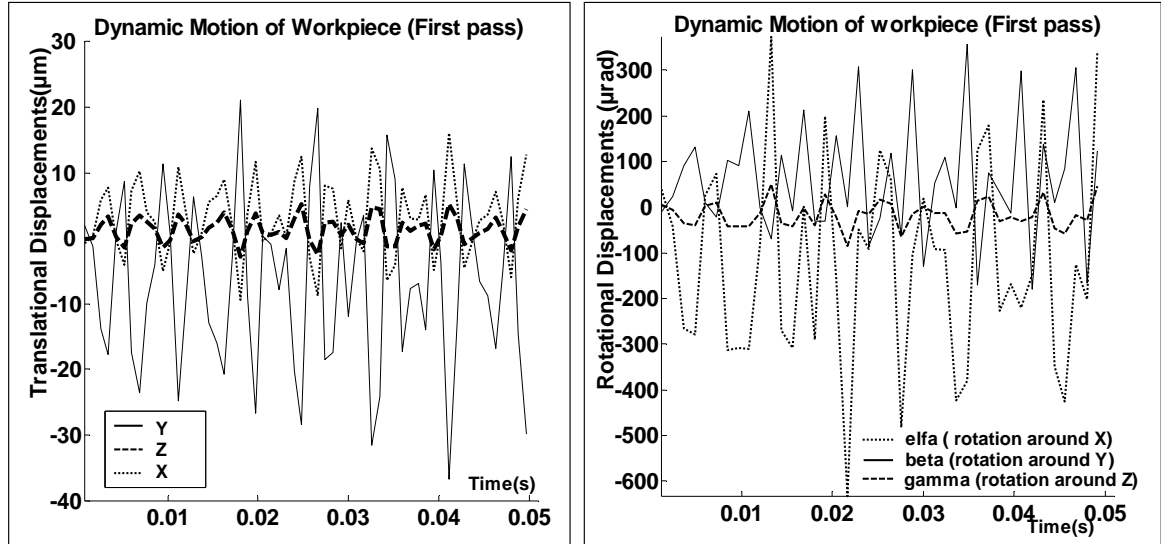


Figure 9: Dynamic motion during the first pass.

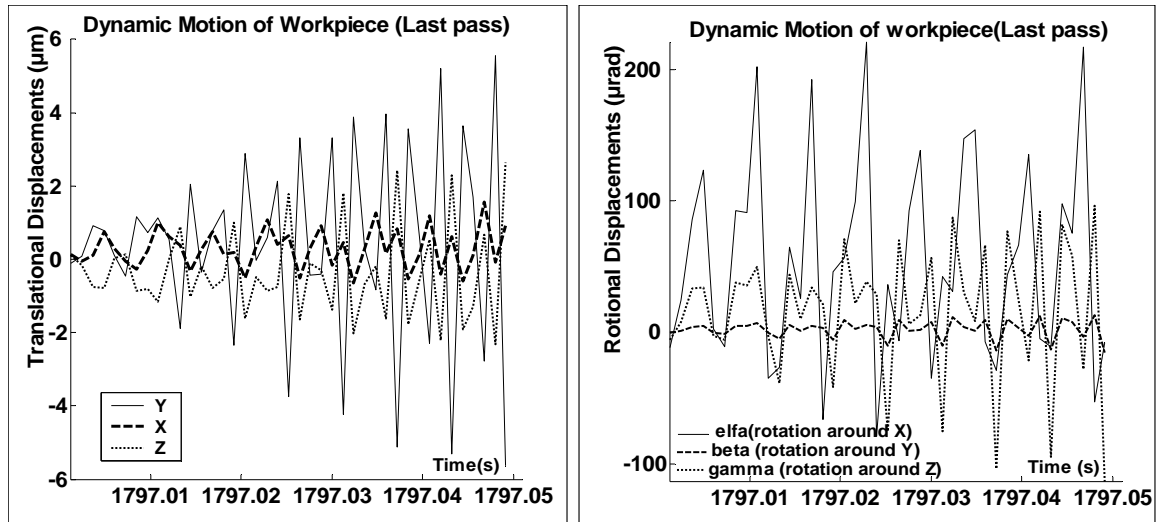


Figure 10: Dynamic motion during the last pass.

This simulation example involving an end milling operation was given to illustrate the theoretical models and stability check procedure. From the results of the example, the key findings are as follows:

- The fixture-workpiece system during the end milling operation presents significant dynamics when certain spindle speeds are used such that the excitation frequency is in the vicinity of a natural frequency of the system. In this scenario, consideration of the fixture-workpiece system dynamics is critical for an accurate analysis of the system.
- Material removal in machining continuously changes the properties of the fixture-workpiece system, e.g., inertia and geometry. When a large portion of material is removed from the workpiece (43% in the simulation example), the fixture-workpiece system behaves quite differently (in the example, the amplitudes of workpiece vibrations were found to be higher during the first tool pass than during the last pass). As a result, higher clamping forces are required to stabilize the workpiece during the first pass than during the last pass.
- Because of the material removal effect, dynamic clamping is an option to achieve the best possible performance of a machining fixture-workpiece system. In addition, allowing different forces at different clamps with a good combination of the clamping forces can improve the overall fixture performance.

## **7. CONCLUSION**

A systematic procedure for analysis of the fixturing dynamic stability of a fixture-workpiece system in machining has been established in this paper. The criteria for fixturing dynamic stability were defined first with lift-off and macro-slip identified as the two types of fixturing instabilities that should be eliminated via proper fixture design. A dynamic model was developed to simulate the vibratory motion of the workpiece during machining and to calculate the resulting dynamic displacements of the workpiece at the fixture-workpiece contacts. A static model was developed to find the contact elastic deformations. The contact dynamic displacements obtained from the dynamic model were superposed with the contact elastic deformations to compute the total contact displacements under the combined effect of clamping and machining. Then the fixturing dynamic stability criteria were applied to detect fixturing instabilities (lift-off and macro-slip) during machining. Through this study, it is seen that:

- Consideration of system dynamics is important for analyzing the fixturing stability in machining operations where the excitation frequencies are in the vicinity of the systems natural frequency.
- Chip removal significantly affects the dynamic behaviour and the fixturing stability when a large percentage of volume removal is involved.
- Satisfaction of fixturing stability criteria requires good combinations of clamping forces, which can be determined from the model and approach developed in this paper.

Future research will focus on the consideration of workpiece structure compliance for relatively flexible parts and the optimization of the clamping forces.

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