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STABILITY ANALYSIS OF A BORING PROCESS UNDER REGENERATIVE CUTTING CONDITIONS

Bourdim, A.*; Bourdim, M.*; Metalsi, F.*; Abdelkader, M. R. H.**
*Faculté des Sciences de l'Ingénieur, Université Aboubekr Belkaid Tlemcen
B.P.230-13000 – Tlemcen, Algérie
bourdim@mail.univ-tlemcen.dz, bourdim56@yahoo.fr,
**University of Jordan, Faculty of Engineering &Technology,
P.O.Box 13258 – Amman – JORDAN
e-mail: abdlkadr@ju.edu.jo

Abstract:
This paper presents a theoretical and experimental investigation into the stability of a cantilever boring bar under regenerative cutting conditions. As this cutting tool is flexible, the regenerative chatter variation often occurs and causes several problems which not only limits productivity of cutting process, but also affects surface finish, cause premature tool failure and reduce dimensional accuracy of the machined part. Its prediction is then important as a guidance to the machine tool user for an optimal selection of cutting conditions, resulting in maximum chip removal rate without this undesirable self excited vibration.

While boring, the chatter vibration may occur in the through-thickness direction of workpiece. The vibration changes removal cross section of workpiece in the present cut and furthermore in the next cut. This change in cross section produces dynamic cutting force variation. When this dynamic cutting force excites the mechanical structure and grows up the previous vibration, this closed-loop instability leads to the regenerative chatter vibration.

The boring bar is modeled at the tool point by a mass, spring and damper system free to move in the two mutually perpendicular directions. The solution of the non uniform equilibrium equation of motion of the cutting tool (boring bar) yielded a characteristic equation in a form of a fourth order polynomial with complex and variable coefficients. The stability of this complex polynomial is based on the Nyquist criterion. This allowed us to plot graphs of stability which indicated clearly that the cutting condition has a decisive influence on the generation of chatter vibrations leading to the instability of cutting. The developed model is verified by cutting experiments, and it is expected that the computed results are in good agreement with the experimental one, and the analytical model is useful to optimize the cutting conditions for highly efficient cutting process.

Key Words: Boring Process, Dynamic Stability, Regenerative Chatter Vibration, Cutting Width, Cutting Speed

1. INTRODUCTION

The machining of metal is often accompanied by a violent vibration of workpiece and cutting tool known as chatter. It is recognised as self excited vibration generated in a closed loop system by variation of cutting forces caused by cutting process itself. It is a basic performance limitation of machine-tool, and affects surface finish, dimensional accuracy, tool life and machine-tool life [1], [2].

Therefore, for a given machining process, it is of great importance to have prior information of the conditions that lead to cutting instability and chatter.

The main concern of the present work is to predict both analytically and experimentally the onset of chatter vibration in boring operation, and to determine the effects of the cutting conditions used on its dynamic behaviour.

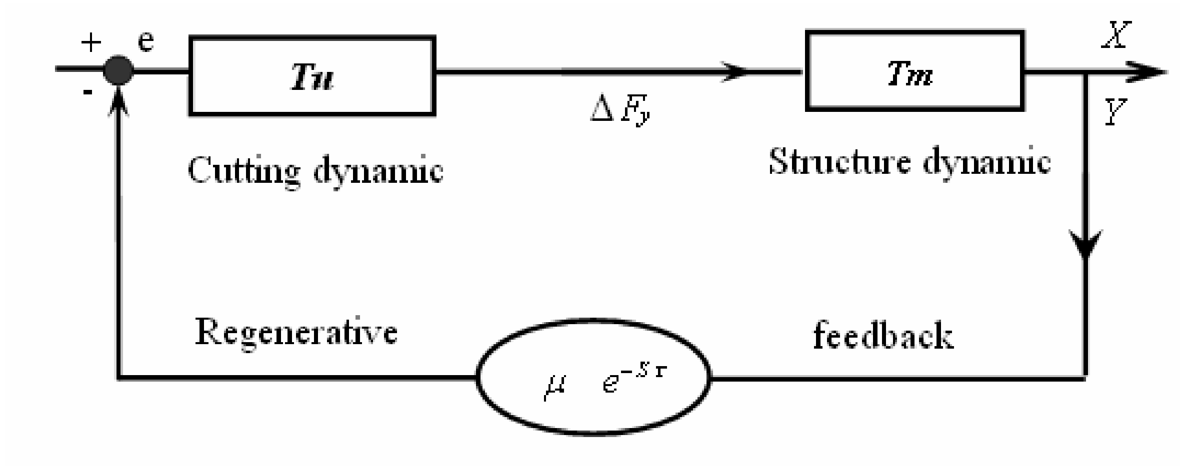

Figure 1: Block diagram for regenerative chatter.

This phenomenon has been analysed by combination of cutting process dynamic and the mechanical receptance of the boring bar in a closed feedback loop that instantaneously controls the tool-workpiece motion (Figure 1).

2. SELF-EXCITED VIBRATION OF A BORING BAR

2.1 Differential equations of the motion of a boring bar

The authors have previously pointed out that the vibrations of a cutting tool while machining are considered to be bi-directional, because the tip tool trajectory is of elliptic form (Figure 2). The structural dynamics of the system is then represented by an equivalent two degrees of freedom (m, c, λ). The cutting force components are assumed to be function of both chip thickness and its displacement velocity ant the interaction of the other component. Otherwise, the two components are coupled [3], [4].

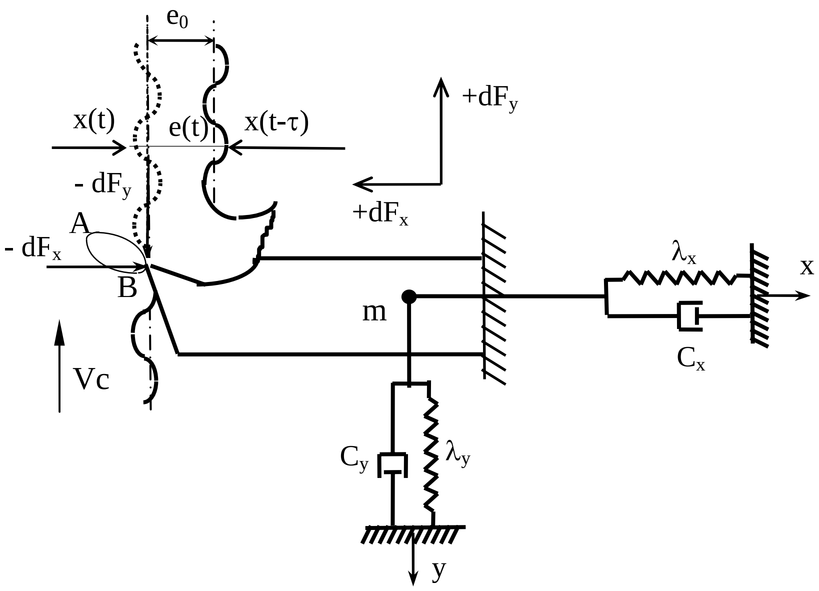

Figure 2: Two degrees of freedom model of the boring bar while machining.

$$dF_x = -k_1\left(x(t) - \mu\, x(t-\tau)\right) + \frac{2\pi}{\Omega} k\, x'(t) + k_{xy}\, dF_y \tag{1}$$

$$dF_y = b\left[-k_1'(x(t) - \mu x(t-\tau) + \frac{2\pi}{\Omega}\nu k' x©(t) + \left(\frac{k_\Omega'}{R} - k'\frac{e_0}{R\Omega}\right) y'(t) + k_{xy} dF_x \right] \tag{2}$$

The differential equations of motion for such system under regenerative chatter conditions may be derived by reference to figure 2 as:

$$m x'' + c x' + \lambda\, x = -dF_x \tag{3}$$

$$m'\, y'' + c'\, y' + \lambda'\, y = -dF_y \tag{4}$$

Substituting:

1: Equations *(1)* and *(2)* into *(3)*, it becomes:

$$m\, x'' + c\, x' + \lambda\, x = k_1\left(x(t) - \mu\, x(t-\tau)\right) - \frac{2\pi}{\Omega} k\, x'(t) - k_{xy}\left(m'\, y'' + c'\, y' + \lambda'\, y\right) \tag{5}$$

Taking its Laplace transform, we find:

$$\left(m S^2 + \left(c + \frac{2\pi}{\Omega} k\right) S + \left(\lambda - k_1\left(1 - \mu e^{-S\tau}\right)\right)\right) X(s) = -k_{xy}\left(m'\, S^2 + c'\, S + \lambda'\right) Y(s) \tag{6}$$

2: Equations *(2)* and *(3)* into *(4)* it becomes:

$$m'\, y'' + c'\, y' + \lambda'\, y = b\begin{bmatrix} k_1'(x(t) - \mu x(t-\tau) - \frac{2\pi}{\Omega}\nu k'\, x'(t) - \left(\frac{k_\Omega'}{R} - k'\frac{e_0}{R\Omega}\right) y'(t) - \\ - k_{xy}\left(m\, x'' + c\, x' + \lambda\, x\right) \end{bmatrix} \tag{7}$$

Taking its Laplace transform, we find:

$$\begin{aligned} &\left(m'\, S^2 + \left[c' + \frac{2\pi}{\Omega} bk' + b\left(\frac{k_\Omega'}{R} - k'\frac{e_0}{R\Omega}\right)\right] S + \lambda' - bk_1'\left(1 - \mu e^{-S\tau}\right)\right) Y(s) = \\ &= bk_{xy}\left(m S^2 + c\, S + \lambda\right) X(s) \end{aligned} \tag{8}$$

Matrix form of *(6)* et *(8)* is given by :

$$\begin{bmatrix} mS^2 + AS + B & -k_{xy}\left(m'S^2 + c'S + \lambda'\right) \\ b\, k_{yx}\left(mS^2 + cS + \lambda\right) & m'\, S^2 + A'\, S + B' \end{bmatrix} \begin{bmatrix} X(s) \\ Y(s) \end{bmatrix} = 0$$

With

$$A = c + \frac{2\pi}{\Omega} k \;;\quad B = \lambda - k_1\left(1 - \mu e_{-S\tau}\right);\quad A' = c' + \frac{2\pi}{\Omega} bk' + b\left(\frac{k'_\Omega}{R} - k'\frac{e_0}{R\Omega}\right);\quad B' = \lambda' - bk'_1\left(1 - \mu^{e_{-S\tau}}\right)$$

Solution of this homogenous system exists only if the determinant is equal to zero. The characteristic equation of the system under consideration can then be obtained by equating the determinant of the above matrix to zero.

$$\left[mS^2+\left(c+\frac{2\pi}{\Omega}k\right)+\left(\lambda-k_1\left(1-\mu e^{-S\tau}\right)\right)\right]\left[mS^2+\left(c'+\frac{2\pi}{\Omega}bk'+b\left(\frac{k'\Omega}{R}-k'\frac{e_0}{R\Omega}\right)\right)S+\lambda'-bk'_1\left(1-\mu e^{-S\tau}\right)\right]-$$
$$-\left[\left(k_{xy}\left(m'S^2+c'S+\lambda'\right)\right)\left(bk_{xy}\left(mS^2+cS+\lambda\right)\right)\right]=0 \qquad (9)$$

Expanding this equation, we obtain a forth order polynomial equation in S, with complex and variable coefficients as:

$$G(s) = A(s)S^4 + B(s)S^3 + C(s)S^2 + D(s)S + E(s)=0 \qquad (10)$$

Substituting S by $j\omega$, with $\alpha_1 = 0$ *into (10)*, equation $G(j\omega)$ can be written in order to separate the reel $\Re_e$ et imaginary $\Im_m$ parts.

$$G(j\omega) = \Re_e(G(j\omega)) + \Im_m(G(j\omega))$$

2.2 Stability charts of the boring bar system (stability criterion)

According to the Nyquist criterion, the dynamic system under consideration will be stable only if the characteristic amplitude-phase-frequency of polynomial characterizing the transfer function of the system will not intercept the real axis in the interval $(-\infty,-1)$ [5], [6], [7].

Figures 3 and 4 represent in the complex plane, the polar diagrams of the dynamic system for ∞ ranging from 0 to $-\infty$ for different values of the width of the cut b and the angular velocity N.

In each case we have draw three different curves characterizing the three different behaviours of the vibrating system.

- The characteristic curve *(1)* does not intercept the real axis in the interval $(-\infty,-1)$: the system is then stable.
- The system is in the limit of the stability when the characteristic curve *(2)* is tangent to the negative real axis, or when it intercepts with it at the critical point *(0,-1)*.
- The characteristic curve *(3)* intercepts the real axis in the interval $(-\infty,-1)$: the system is then unstable.

As it can be seen from the analysis of the characteristic equation of the system, the stability depends on the machining conditions and the dynamic characteristics of the rotating dynamic system. The stability of the system will therefore be assured only if:

$\Re e\,(G(j\omega))>-1$ and $\Im m\,(G(j\omega))= 0$ in the interval $(0<\omega<+\infty)$

As it is shown by Figure 3 and 4, the width of the cut has a significant influence on the behaviour of the system during the machining operation. For an angular velocity *N =200 rpm*, the threshold of stability is occurred for a cutting width equal to a bout 1mm and is independent of natural frequency of the bar. However for values of *b=2.5mm*, the cut becomes more stable for smaller cutting speed. But the value of N at which the threshold of chatter occurs depends on natural frequency as shown by figure 5 and 6. This phenomenon is probably due to the damping forces whose influence on the stability of the cut at high speeds is much more important than the width of the cut [8], [9].

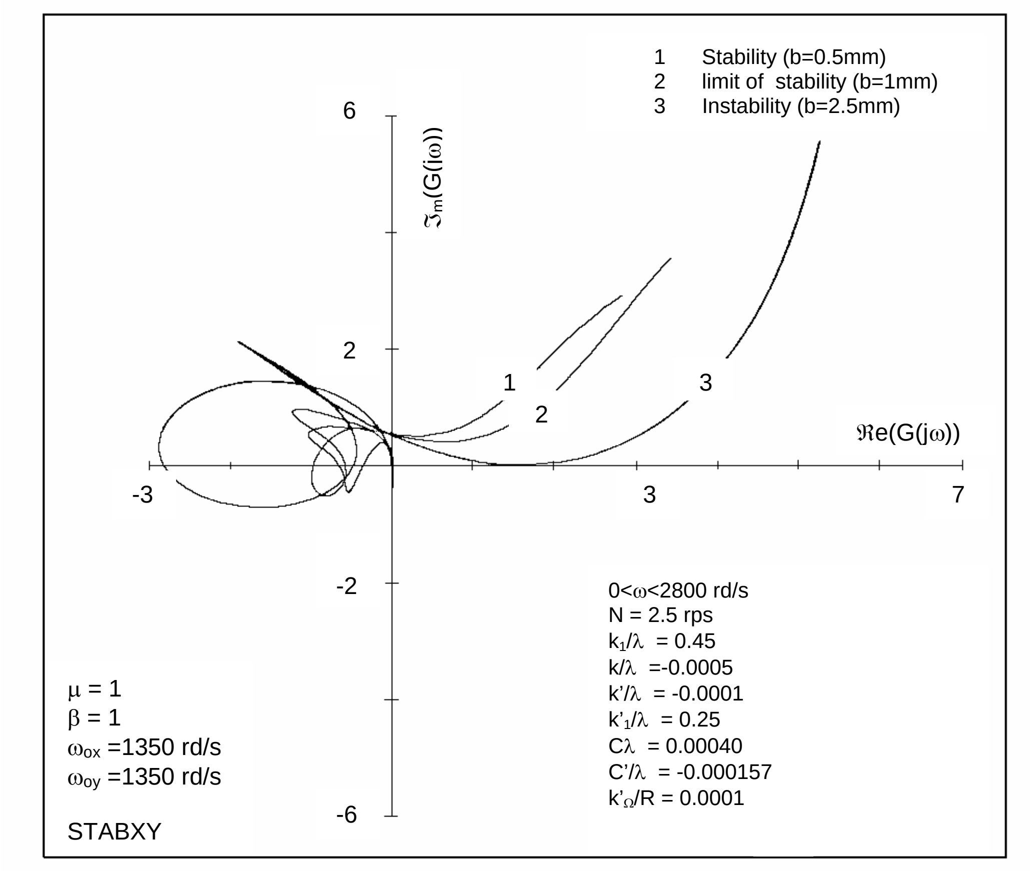

Figure 3: Polar graph (*b* variable).

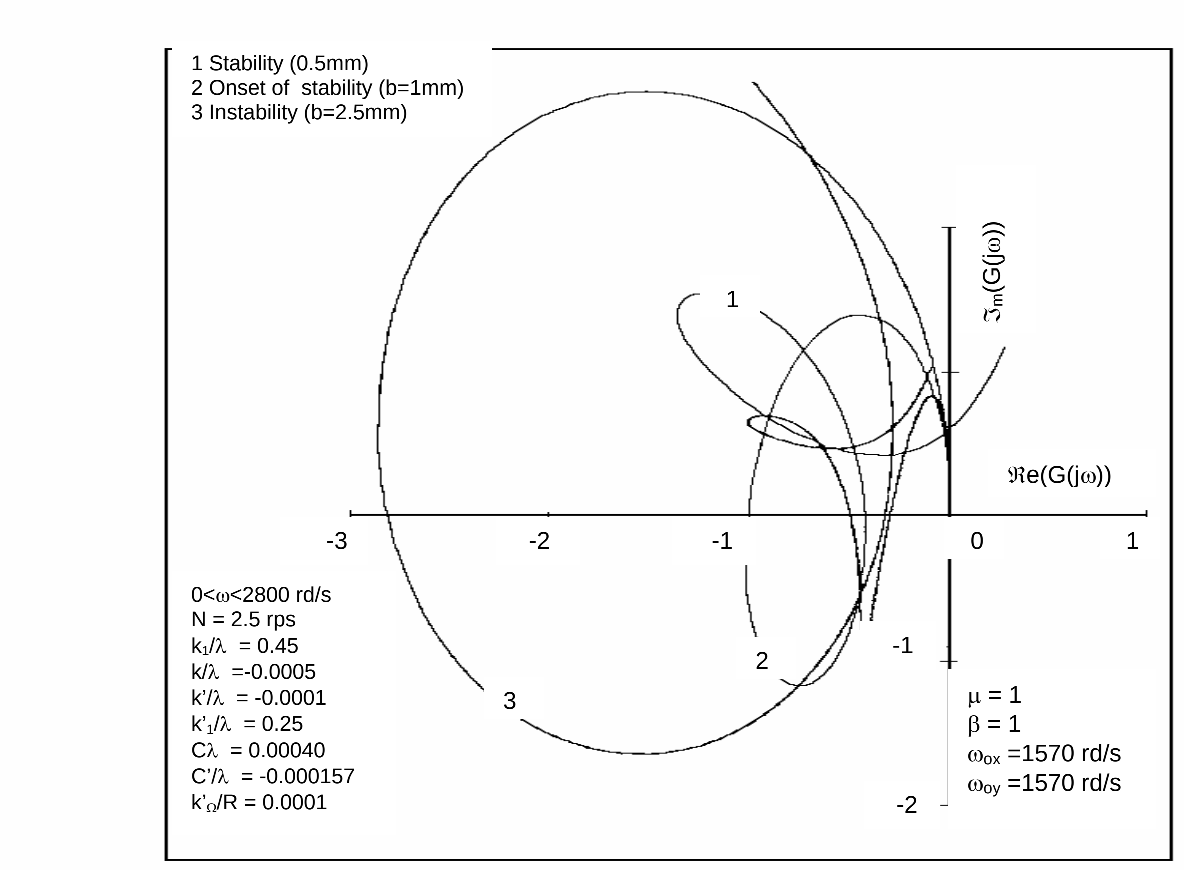

Figure 4: Polar graph (*b* variable).

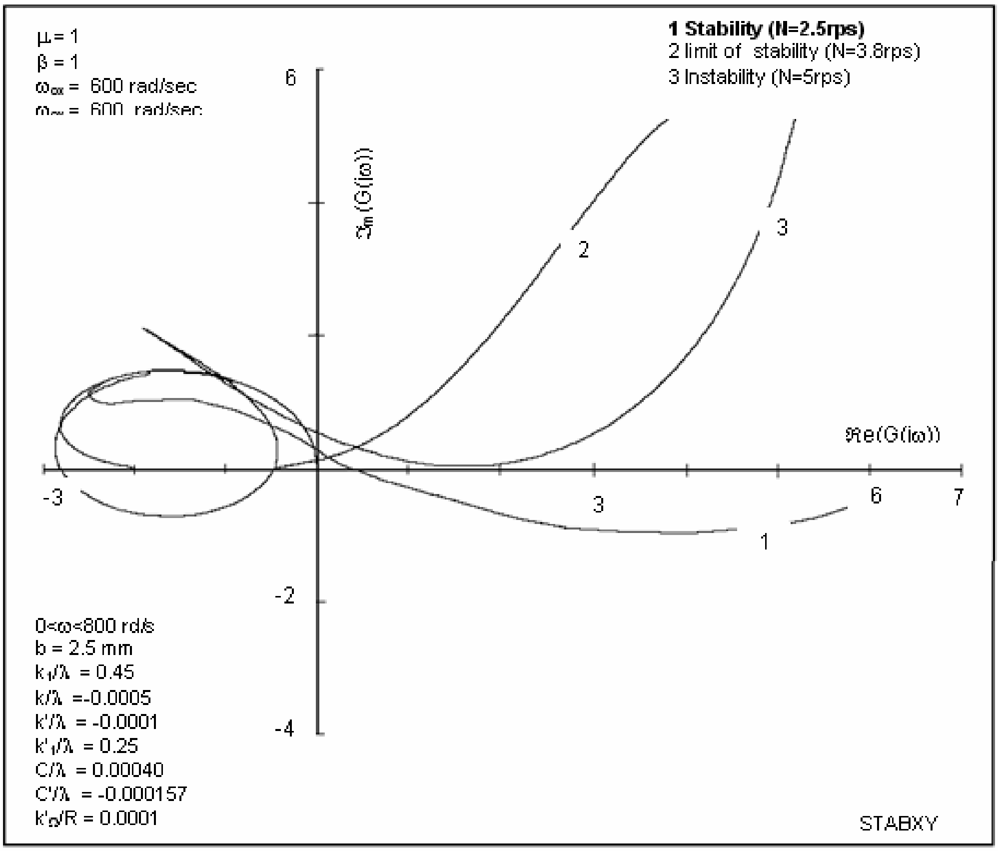

Figure 5: Polar graph (*N* variable).

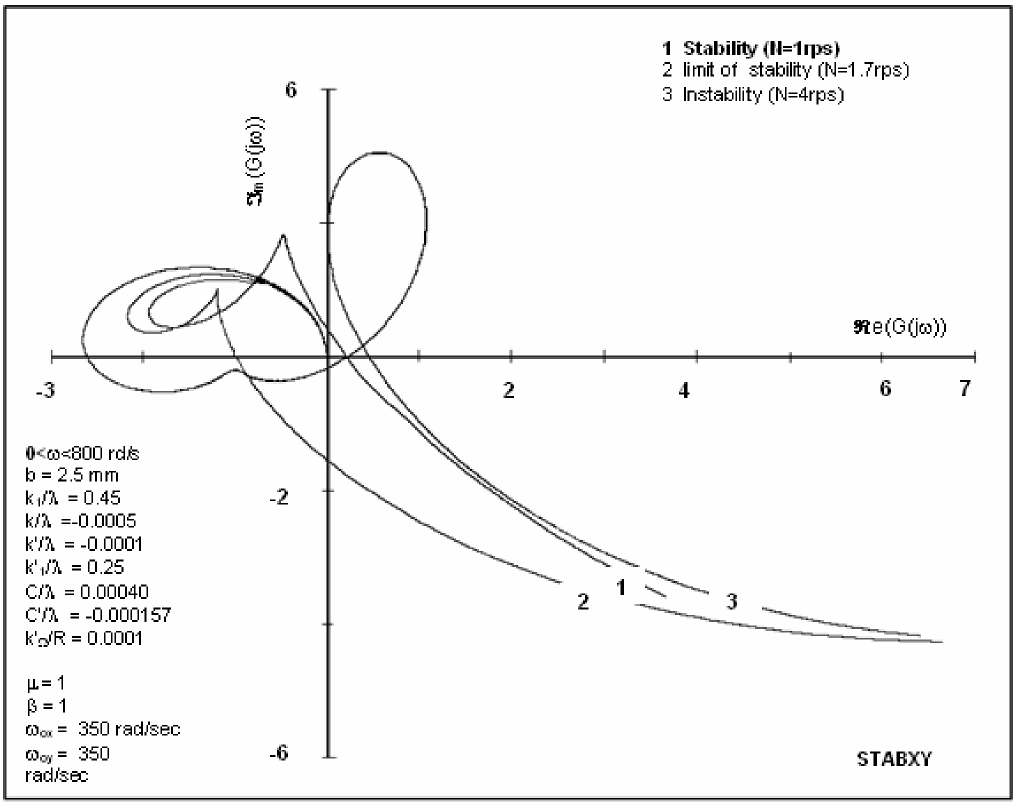

Figure 6: Polar graph (*N* variable).

3. EXPERIMENTAL RESULTS

3.1 Experimental equipment:

- **Workpiece :** Tapered test pieces were used in order to obtain a continuous increase of the depth of cut (*5%* of conicity, 135*mm* of diameter and 50*mm* of thickness). The workpiece is rigidly linked to the machine-tools.
- **Cutting tool:** Centered cutting tool “sandvik” type (TmaxP CNMG 120412) is used and rigidly fixed on a circular boring bar fixed with an overhang.
- **Displacement measurement:** Proximity transducer, Bently Névada type.
- **Analysis equipment:** This equipment is constituted by a signal analyzer (HP 3562A) and a memory drawing table (HP 7090A).

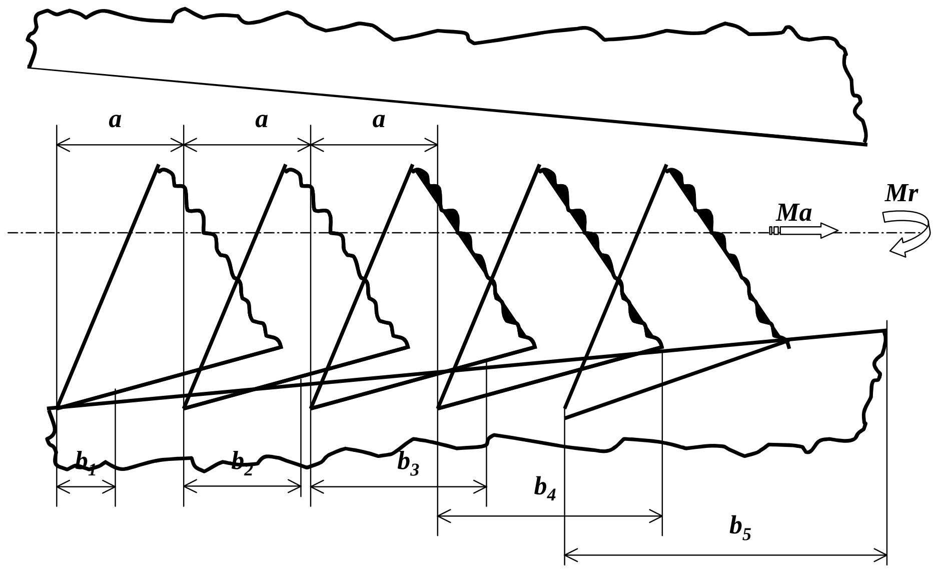

Figure 7: Variation cutting width.

3.2 Results and discussion:

As it can be seen in figure 8a and b, the amplitude of the chatter vibration increase with b. Nevertheless there exists critical cutting width at which sudden chatter vibrations occur. The variation of vertical vibration amplitude is more significant than that of horizontal vibration.

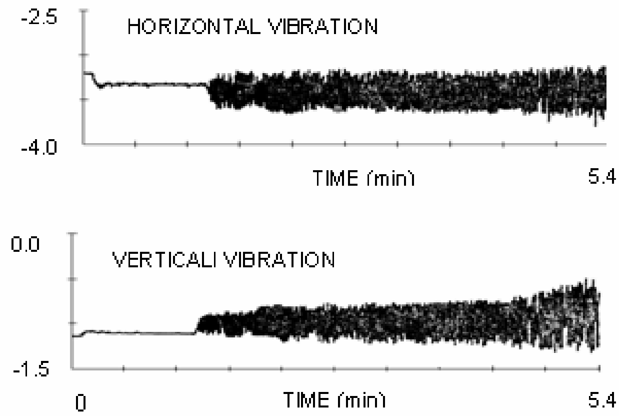

Figure 8a: Effect of Continuous variation of cutting width on chatter vibration.

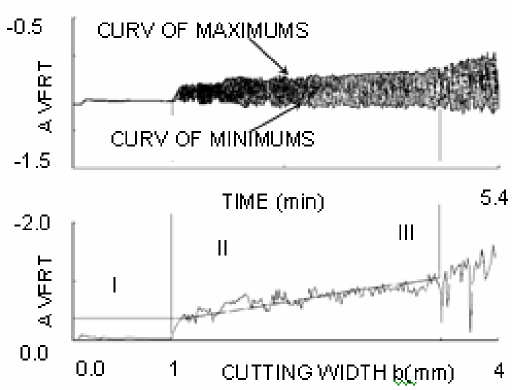

Figure 8b: Variation of amplitude of chatter vibration with cutting width.

4. CONCLUSION

Dynamic instability of the boring operation is a regenerative type. The analysis carried out to predict the regenerative instability has been confirmed.
The theoretical analysis presented in this work for predicting the conditions of instability has been verified by the good agreement obtained with experimental results.
The optimal cutting widths, for a stable machining, depend on the used cutting speed. At low cutting speeds, it has been found that the instability occurs at higher widths of cut. However, at higher cutting speeds, the instability may occur at low widths of cut.

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