

## Effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework

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### ABSTRACT

This study explores the effect of delayed differentiation on a multiproduct vendor-buyer integrated inventory system with rework to identify its potential benefits and provide managers with in-depth information for operational decision-making. The main considerations of the proposed study include a multiproduct fabrication plan to increase machine utilization, a rework process to ensure product quality, and a multi-shipment policy to distribute the end products. In addition, these products sharing an intermediate part for which a two-stage fabrication scheme is adopted, wherein the common parts are produced at the first stage and the end products are manufactured at the second stage. The aim is to reduce the overall system costs and shorten the replenishment cycle time. Mathematical modeling and optimization methods were employed to derive the closed-form optimal replenishment cycle time and delivery decisions. We demonstrated the applicability of our research results through numerical examples and revealed that for both linear and nonlinear relationships between the common intermediate part's completion rate  $\alpha$  and its practical value at  $\alpha$ , our proposed two-stage production scheme with delayed differentiation is considerably beneficial vis-à-vis single-stage schemes in saving overall system costs and reducing the replenishment cycle time.

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## Vpliv zakasnjenega razlikovanja na večizdelčni integrirani inventarni sistem prodajalec-kupec s predelavo

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### POVZETEK

Študija raziskuje vpliv zakasnjenega razlikovanja na večizdelčni prodajalec-kupec integrirani inventarni sistem s predelavo, z namenom identifikacije potencialnih prednosti takega sistema, kakor tudi ponuditi menedžerjem poglobljene informacije za sprejemanje delovnih odločitev. Študija vključuje načrt za povečanje izrabe strojev pri izdelavi različnih izdelkov, predelovalni proces za zagotovitev kakovosti izdelkov, in politiko večstopenskega odpošiljanja za zagotovitev distribuirane odpreme končnih izdelkov. Ker so različni končni izdelki sestavljeni iz enakih sestavnih delov se izdelajo dvostopenjsko. V prvi fazi se izdelajo sestavni deli, v drugi fazi pa končni izdelki. Cilj študije je zmanjšanje celotnega stroška in skrajšanje časa do prodaje. Za izpeljavo optimalnega časa do prodaje in odločitve o dostavi zaprtrega tipa so uporabljeni matematični modeli in optimizacijske metode. Uporabnost raziskave je potrjena z računskimi primeri, pokaže pa se tudi, da je za linearno in nelinearno odvisnost med stopnjo izdelave sestavnih delov  $\alpha$  in njeno cenovno vrednostjo pri  $\alpha$ , pri prihranku celotnega stroška in skrajšanju časa odprodaje predlagana dvostopenjska produkcijska shema z zakasnjenim razlikovanjem boljša od enostopenjskih shem.

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### PODATKI O ČLANKU

*Ključne besede:*  
Večizdelčni prodajalec-kupec sistem  
Odločitev izdelava-odprodaja  
Predelava  
Enaki sestavnii deli  
Zakasnjeno razlikovanje

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