

## Time-dependent and bi-objective vehicle routing problem with time windows

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### ABSTRACT

The optimization of bi-objective vehicle routing problem has become a research hotspot in recent years. In this paper, a time-dependent and bi-objective vehicle routing problem with time windows (TD-BO-VRPTW) is proposed, which is a new extension of classical vehicle routing problem. Time-dependency is presented for the situation that vehicle's travel speed is affected by its departure time and the distance between two customers. The total transportation costs and time costs are two objectives optimized simultaneously through constructing a bi-objective mixed integer linear programming model. To deal with this problem, the non-dominated sorting genetic algorithm II (NSGA-II) is adopted to obtain the Pareto optimal solution set. In the numerical examples, the RC108 from Solomon's benchmark set is employed and the results in the Pareto front show the efficiency of NSGA-II for the TD-BO-VRPTW. To further test the performance of this algorithm, two objectives are optimized separately and then the sum of two objectives is also optimized. Through comparing these results with solutions in the Pareto front, it can be concluded that the algorithm is reliable, and the results in Pareto front are competitive because there is a trade-off between two objectives.

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# Časovno odvisen in dvokriterijski problem usmerjanja vozil s časovnimi okni

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## P O V Z E T K

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Dvokriterijska optimizacija usmerjanja vozil je v zadnjih letih postala vroča raziskovalna tematika. V tem prispevku je predlagan časovno odvisen in dvokriterijski problem usmerjanja vozil s časovnimi okni (TD-BO-VRPTW), kar je nova razširitev klasičnega problema usmerjanja vozil. Predstavljena je časovna odvisnost za situacije, pri katerih na hitrost vožnje vozila vplivata čas odhoda in razdalja med dvema strankama. Skupni stroški prevoza in časovni stroški sta dva kriterija, ki sta istočasno optimizirana z izgradnjo dvokriterijskega modela mešanega celoštivilskega linearnega programiranja. Za reševanje tega problema je uporabljen genetski algoritmom z nedominiranim razvrščanjem II (NSGA-II), s katerim dobimo nabor Pareto optimalnih rešitev. V številčnih primerih je uporabljen primer RC108 iz Solomonovih standardnih testnih primerov, rezultati Pareto fronte pa kažejo na učinkovitost NSGA-II za TD-BO-VRPTW. Za preverjanje uspešnosti NSGA-II algoritma se oba kriterija optimizirata ločeno, nato pa je optimizirana še vsota obeh kriterijev. Če primerjamo te rezultate z rešitvami na Pareto fronti, je mogoče sklepati, da je algoritmom zanesljiv, rezultati na Pareto fronti pa so konkurenčni, saj je prišlo do kompromisne rešitve med dvema ciljema.

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## P O D A T K I O Č L A N K U

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*Ključne besede:*  
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Časovna odvisnost;  
Dvokriterijska optimizacija;  
Časovna okna;  
Optimalne rešitve Pareto;  
Evolucijski algoritmi;  
Algoritem NSGA-II

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