Manufacturer’s customer satisfaction incentive plan for duopoly retailers with Cournot or collusion games

Hu, H., Zhang, Z., Wu, Q., Han, S.

School of Economics and Management, Yanshan University, Qinhuangdao, P.R. China
Research Center for Regional Economic Development, Yanshan University, Qinhuangdao, P.R. China

ABSTRACT

To increase customer satisfaction (CS) which is closely linked to corporate reputation, revenue and customer loyalty, manufacturers will provide incentives to retailers in supply chain management. This paper focuses on two types of incentives that a manufacturer may provide to retailers: customer satisfaction index bonus (CSI bonus) and customer satisfaction assistance, and studies the optimal customer satisfaction incentive plan of the manufacturer when duopoly retailers adopt Cournot or collusion game. By comparing the equilibrium of the two games, we conducted a preference analysis of both the manufacturer and the retailers. The results showed that no matter what kind of games the duopoly retailers take, the manufacturer will provide customer satisfaction assistance to the retailers to increase the customer satisfaction. However, if the duopoly retailers take Cournot behaviour, only when the wholesale price is greater than the threshold, the manufacturer will provide the retailers with customer satisfaction index bonus. The manufacturer always prefers to Cournot behaviour, and the retailers always prefer to collusion behaviour. In addition, this paper also investigated the effect of customer satisfaction incentives on the manufacturer, and found that it will help the manufacturer obtain more demand and higher profits.

ARTICLE INFO

Keywords:
Supply chain management;
Manufacturer’s incentive plan;
Customer satisfaction;
Duopoly retailers;
Game;
Cournot game;
Collusion

*Corresponding author: huhaiju@ysu.edu.cn (Hu, H.)

Article history:
Received 12 August 2020
Revised 23 September 2020
Accepted 29 September 2020

© 2020 CPE, University of Maribor. All rights reserved.

1. Introduction

Nowadays, CS is concerned by various industries, so it is imperative to improve it. CS plays such an importance role because it is closely related to a company’s reputation, marketing activities and revenues [1-2]. High CS will result in more repeat purchases, cross purchases and a good reputation [3-5]. When CS is low, it will make consumers have a bad impression of the company, which will affect the reputation of the company and make it difficult for the company to last for a long-term development. Therefore, to encourage retailers to improve CS, many manufacturers implement some form of supply chain incentive measures. In practice, manufacturers typically offer two types of CS incentives to retailers: CSI bonus and CS assistance [6-7]. Specifically, CSI bonuses are one-time incentives given by manufacturers to retailers based on CSI scores. According to the 2017 CSI automotive industry research results in China, the CSI scores of Audi (86.6), BMW (80.1) and Mercedes-Benz (71.5) are the top three in the luxury car series; Toyota (81.8), Chevrolet (77.8) and Volkswagen (73.1) are listed as the top three in the mainstream car series. They gave retailers a certain CSI reward based on CSI scores. CS assistance is an investment made by manufacturers to share the cost of retailers’ efforts to improve CS. For example, Ford and Lexus train the employees of investment retailers for free to reduce the input cost of retailers’ CS efforts; Benz and Saturn offer free consulting services to retailers to improve CS.
CS was first proposed by Cardozo [8], who argued that improving CS would lead to a repurchasing behaviour. Wang and Zhao [1] established a multilayer linear model discussed the impact of CS on the value of shareholder. There are also many studies to improve CS by implementing some kinds of incentive. The CSI was first established by Sweden in 1989. It is an index calculated based on customer evaluation of enterprise product and service quality. Hauser et al. [9] were the first to study CSI bonuses, and they considered a two-phase supply chain model, in which the CS incentive plan increased the future demand of the second phase. Based on this, Chu and Desai [6] proposed another incentive: CS assistance, which encourages retailers to improve CS, and they found that the manufacturer would provide these two incentives in their CS incentive plan when the actors all have strong pricing power. Wang et al. [7] studied the optimal CS incentive plan for manufacturers in the four supply chain models, and they further found that when the manufacturer’s market pricing ability is weak and the wholesale price is lower than a certain value, the manufacturer only provides CS assistance to the retailers.

These literatures mainly study that a manufacturer provide incentives to a retailer to improve CS, but the situation of providing incentives to duopoly retailers has not been studied. Generally speaking, there are many duopoly supply chains, such as offline sales Gome and Suning, Wal-Mart and Tesco, and online sales such as JD.com and Tmall. By cooperating with duopoly retailers, the manufacturer can quickly expand their market influence, and it must also improve CS in the face of fierce competition between similar products, so as to win in the competition. There are different forms of competition among duopoly retailers. Competition between duopoly retailers is a common phenomenon in various industries. They (e.g. Wal-Mart and Tesco) determine their own selling price and order quantity respectively and pursue their own profit maximization. Cooperation is another behaviour between duopoly retailers, who jointly set their selling price so that the entire downstream supply chain of profit maximization. Yang and Zhou [10] studied the three behaviours of the duopoly retailers Cournot, collusion and Stackelberg, and conducted preference analysis based on the best decision. They finally found that if the duopoly retailers take collusion behaviour, its selling price is the highest; However, if the duopoly retailers take Cournot behaviour, its selling price is the lowest. Huang et al. [11] established six decentralized models and a centralized model according to different rights structures. Modak et al. [12] studied the Cournot and collusion model of duopoly retailers in a closed-loop supply chain. Herbon [13] studied the price, profit and market share of two competition retailers in the context of consumer information asymmetry. Li et al. [14] studied a combination of strategies of one manufacturer and multiple suppliers for cooperative supply. Lang and Shao [15] studied the product portfolio decisions of two retailers under competition situations based on the MNL model. Chai et al. [16] studied the price competition of two competing retailers under stochastic demand disruption.

In supply chain coordination, there are many coordination contracts, such as volume discount contracts [17], two-part tariff contracts [18-19], revenue-sharing contracts [20-22], incentive adjustment policy [23], cost sharing contracts [24], etc. Hu et al. [25] studied the competition among DCSC members and the optimal strategy in these two coordination situations. This paper focuses on improving supply chain performance through incentive policies. Cheng et al. [26] explored incentive contracts under conditions of uncertain demand and asymmetric information. Deng et al. [27] studied the incentive mechanism of CS in the distributed service chain. Hu et al. [28] studied the retailers provide incentives for manufacturers to improve product quality, thereby increasing consumers’ willingness to purchase. Deng et al. [29] designed an effective CS incentive contract taking budget constraints into account, and they found that even in a single period environment that the customers would not bring in future business, CS incentives benefit the manufacturer.

This paper builds 2 kinds of game scenarios of the duopoly retailers. The specific arrangements are as follows: Section 2 describes the model. Section 3 studies the manufacturer's CS incentive plan in two game scenarios, and compares and analyses the decision results. Section 4 studies the impact of CS incentives on the manufacturer. Section 5 summarizes conclusions, points out deficiencies and looks to the future.
2. Model description

This study focuses on the supply chain consisting of one manufacturer and two retailers. The manufacturer wholesales the same product to the two retailers at the same price, and then retailers sell the product to consumers in the same market.

The demand function is based on Chu and Desai [6] which merges short-term demand and long-term demand into a single demand function, and add to this demand function the effect of another retailer’s price, resulting in a new demand function as follows.

\[ D_i = \alpha_i - p_i + b_i + \beta p_i (3-i) \quad (i = 1,2) \]  

\( D_i \): Retailer i’s demand function;  
\( \alpha_i \): Retailer i’s market potential (initial reputation);  
\( p_i \): Retailer i’s selling price;  
\( b_i \): Retailer i’s CS efforts;  
\( \beta \): The degree of substitutability between retailers, reflects the impact of retailer’s marketing mix decisions on customer demand, with \( 0 \leq \beta < 1 \).

Assume retailer i’s CS effort cost is \( b_i^2 \). CS incentive plan consists of two parts: (1) CSI bonus, the bonus per unit CS effort is \( \eta \), CSI bonus is measured by retailer i’s CS effort \( b_i \) direct measurement, the manufacturer gives the retailer i’s the total amount of CSI bonus is \( \eta b_i \). (2) CS assistance, the assistance per unit CS effort is \( x_b \), so the retailer i’s CS effort cost reduction \( b_i x_b \), cost is \( b_i (b_i - x_b) \), then the cost incurred by the manufacturer is \( b_i^2 \).

Retailer i the profit function \( \pi_{ri} \) is:

\[ \pi_{ri} = (p_i - w_m)D_i - b_i (b_i - x_b) + \eta b_i \]  

Where \( w_m \) is the wholesale prices offered by the manufacturer.

Assume that the manufacturer’s manufacturing cost is 0, the manufacturer’s profit function \( \pi_m \) is:

\[ \pi_m = \sum_{i=1}^{2} (w_m D_i - \eta b_i - b_i^2) \]  

3. Model analysis

In the model analysis below, we assume that the manufacturer is the leader of Stackelberg and the duopoly retailers is its follower, and the retailers can play either Cournot or collusion game.

3.1 Retailers play the Cournot game

In this section, it is assumed that duopoly retailers adopt a Cournot approach in which each retailer sets its own selling price and CS effort by assuming that its competitors’ selling price and CS effort.

Our game sequence is:

(1) the manufacturer determines \( w_m, \eta, x_b \);  
(2) retailer i according to the given \( w_m, \eta, x_b \) to determine \( p_i, b_i \).

In order to find the equilibrium of the subgame of the two-stage game, the backward induction method is adopted.

For any given \( w_m, \eta, x_b \), denoting \( \frac{\partial \pi_{ri}}{\partial p_i} = 0, \frac{\partial \pi_{ri}}{\partial b_i} = 0 \) we have the optimal selling price \( p_i^* \) and the best CS effort \( b_i^* \); Retailer 2 passed solve \( \frac{\partial \pi_{r2}}{\partial p_2} = 0, \frac{\partial \pi_{r2}}{\partial b_2} = 0 \) to determine the optimal selling price \( p_2^* \) and the optimal CS effort \( b_2^* \). Followed, putting \( p_1^*, b_1^*, p_2^*, b_2^* \) into equation (3), the manufacturer solves \( \frac{\partial \pi_m}{\partial w_m} = 0, \frac{\partial \pi_m}{\partial \eta} = 0, \frac{\partial \pi_m}{\partial x_b} = 0 \) to determine the optimal wholesale price \( w_m^* \), the optimal CS effort bonus per unit \( \eta^* \), the optimal CS effort assistance per unit \( x_b^* \).

When the profit function is a concave function on its decision variable, the optimal solution can maximize the profit. Therefore, the concaveness of the profit function is tested by calculating the Hessian matrix.
As can be seen from Eq. 1, the Hessian matrix of \( \pi_{r1} \) is:

\[
H_{c1} = \begin{bmatrix}
\frac{\partial^2 \pi_{r1}}{\partial p_1^2} & \frac{\partial^2 \pi_{r1}}{\partial p_1 \partial b_1} \\
\frac{\partial^2 \pi_{r1}}{\partial p_1 \partial b_1} & \frac{\partial^2 \pi_{r1}}{\partial b_1^2}
\end{bmatrix} = \begin{bmatrix}
-2 & 1 \\
1 & -2
\end{bmatrix}
\]

\( \Delta 1 = -2 < 0; \Delta 2 = 3 > 0 \). Retailer 1’s profit function \( \pi_{r1} \) is its decision variable \( p_1 \) with \( b_1 \) strict concave function, so \( \pi_{r1} \) has a maximum value.

As can be seen from Eq. 2, the Hessian matrix of \( \pi_{r2} \) is:

\[
H_{c2} = \begin{bmatrix}
\frac{\partial^2 \pi_{r2}}{\partial p_2^2} & \frac{\partial^2 \pi_{r2}}{\partial p_2 \partial b_2} \\
\frac{\partial^2 \pi_{r2}}{\partial p_2 \partial b_2} & \frac{\partial^2 \pi_{r2}}{\partial b_2^2}
\end{bmatrix} = \begin{bmatrix}
-2 & 1 \\
1 & -2
\end{bmatrix}
\]

\( \Delta 1 = -2 < 0; \Delta 2 = 3 > 0 \). Retailer 2’s profit function \( \pi_{r2} \) is its decision variable \( p_2 \) with \( b_2 \) strict concave function, so \( \pi_{r2} \) has a maximum value.

Bringing \( p_1^{ct}, b_1^{ct}, p_2^{ct}, b_2^{ct} \) in Table 1 to Eq. 3, the Hessian matrix of \( \pi_m \) is:

\[
H_{c3} = \begin{bmatrix}
\frac{\partial^2 \pi_m}{\partial w_m^2} & \frac{\partial^2 \pi_m}{\partial w_m \partial \eta} & \frac{\partial^2 \pi_m}{\partial \eta^2} \\
\frac{\partial^2 \pi_m}{\partial \eta \partial w_m} & \frac{\partial^2 \pi_m}{\partial \eta^2} & \frac{\partial^2 \pi_m}{\partial \eta^2} \\
\frac{\partial^2 \pi_m}{\partial x_b \partial w_m} & \frac{\partial^2 \pi_m}{\partial x_b \partial \eta} & \frac{\partial^2 \pi_m}{\partial x_b^2}
\end{bmatrix} = \begin{bmatrix}
-8\beta + 8 & 2\beta - 4 & -2 \\
2\beta - 4 & -4\beta + 8 & -2\beta + 4 \\
-2 & -2\beta + 4 & -8\beta + 12 \\
2\beta - 3 & 2\beta - 3 & 2\beta - 3
\end{bmatrix}
\]

\( \Delta 1 = -8\beta + 8; \Delta 2 = \frac{4(\beta - 2)(7\beta - 6)}{(2\beta - 3)^2}, \Delta 3 = -\frac{16(\beta - 3)(6\beta - 5)}{(2\beta - 3)^2} \). When \( \Delta 1 < 0, \Delta 2 > 0, \Delta 3 > 0 \), the manufacturer’s profit function \( \pi_m \) is its decision variable \( w_m, \eta, x_b \) strict concave function, only having a maximum value. Making \( \Delta 1 < 0, \Delta 2 > 0, \Delta 3 > 0 \), the solution is obtained \( \beta < \frac{5}{6} \) so we assume \( \beta < \frac{5}{6} \) and the results are shown in Table 1.

When duopoly retailers play the Cournot game, the manufacturer sets a wholesale price threshold \( w_m' \) in order to guarantee its profit. Only when \( w_m > w_m' \), the manufacturer gives the retailers a CSI bonus, otherwise it will not. As can be seen from Table 1 that when \( \beta = 0.25 \), it is a threshold of the wholesale price of the product.

\[
w_m' = \frac{5(\alpha_1 + \alpha_2)}{14}
\] (4)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Equilibrium under the Cournot of duopoly retailers (ct means Cournot)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>wholesale price  ( w_m^{ct} = -\frac{5(\alpha_1 + \alpha_2)}{4(6\beta - 5)} )</td>
</tr>
<tr>
<td></td>
<td>assistance per unit CS effort  ( x_b^{ct} = -\frac{4(\alpha_1 + \alpha_2)}{4(6\beta - 5)} )</td>
</tr>
</tbody>
</table>
|         | bonus per unit CS effort  \( \eta^{ct} = \begin{cases} 
\frac{(4\beta - 1)(\alpha_1 + \alpha_2)}{4(6\beta - 5)(\beta - 2)} & \beta > \frac{1}{4} \\
0 & \beta \leq \frac{1}{4}
\end{cases} \) |
| Stage 2 | retailer 1’s selling price  \( p_1^{ct} = \frac{A(w_m^{ct} + x_b^{ct} + \eta^{ct}) + 2B}{X} \) |
|         | retailer 1’s CS effort  \( b_1^{ct} = \frac{D(w_m^{ct} + E(x_b^{ct} + \eta^{ct}) + B}{X} \) |
|         | retailer 2’s selling price  \( p_2^{ct} = \frac{A(w_m^{ct} + x_b^{ct} + \eta^{ct}) + 2C}{X} \) |
|         | retailer 2’s CS effort  \( b_2^{ct} = \frac{D(w_m^{ct} + E(x_b^{ct} + \eta^{ct}) + C}{X} \) |

Note: \( 0 \leq \beta \leq \frac{5}{6}, X = 4\beta^2 - 9, A = -2\beta - 3, B = -2\beta\alpha_2 - 3\alpha_1, C = -2\beta\alpha_1 - 3\alpha_2, D = -2\beta^2 - \beta + 3, E = 2\beta^2 - \beta - 6 \).
Proposition 1:

1. In the CS incentive plan, if \( \beta > 0.25 \), the manufacturer provides both CSI bonuses and CS assistance; if \( \beta \leq 0.25 \), the manufacturer only provides CS assistance.

2. When the degree of substitutability \( \beta \) between retailers increases, the manufacturer should increase CSI bonuses and CS assistance in the CS incentive plan.

Prove. In order to prove Proposition 1 (2), solving the partial derivative of \( x_b^c \) and \( \eta^c \) with respect to \( \beta \) in Table 1 is needed.

\[
\frac{\partial x_b^c}{\partial \beta} = \frac{24(\alpha_1+\alpha_2)}{(24\beta-20)^2} > 0, \quad \frac{\partial \eta^c}{\partial \beta} = -\frac{(24\beta^2-12\beta-13)(\alpha_1+\alpha_2)}{4(6\beta-5)^2(\beta-2)^2} > 0
\] (5)

Due to \( \frac{\partial x_b^c}{\partial \beta} > 0 \) and \( \frac{\partial \eta^c}{\partial \beta} > 0 \), \( x_b \) and \( \eta \) under Cournot game are increased with \( \beta \), respectively.

Proposition 1 (1) indicates that when \( \beta > 0.25 \), the manufacturer should provide CSI bonuses and CS assistance to incentive retailers to improve CS, when \( \beta \leq 0.25 \), the manufacturer stops to use CSI bonuses. The larger \( \beta \) leads higher wholesale price of the manufacturers. Therefore, manufacturers are willing to provide CSI bonuses to retailers when wholesale prices are high. Conversely, when the wholesale price is low, the manufacturer will not provide the CSI bonus to the retailers. As can be seen from Table 1, \( \beta = 0.25 \) is a threshold for the wholesale price of the product. In addition, retailers alone bear the cost of the CS effort, and the benefits of high CS are shared by both parties. Therefore, manufacturers prefer to provide CS assistance to retailers to reduce the cost of the CS effort, and motivate retailers to make higher CS effort.

Proposition 1 (2) indicates that as \( \beta \) increase, the manufacturer should give retailers more CSI bonuses and CS assistance. Higher substitutability (\( \beta \)) between the two retailers is always preferred by the manufacturer, because the larger the \( \beta \), the more similar the two retailers, and the easier it is for the manufacturer to manage the two retailers.

3.2 Retailers play the collusion game

In this section, it is assumed that duopoly retailers can collude. Therefore, the total profit of retailers is:

\[
\pi_R = \pi_{r1} + \pi_{r2} = (p_1 - w_m)D_1 - b_1(b_1 - x_b) + \eta b_1 + (p_2 - w_m)D_2 - b_2(b_2 - x_b) + \eta b_2
\] (6)

Game sequence is:

1. The manufacturer determines \( w_m, \eta, x_b \);
2. Retailer 1 and retailer 2 determine \( p_i, b_i \) together, according to the given \( w_m, \eta, x_b \).

In order to find the equilibrium of the subgame of the two-stage game, the backward induction method is adopted. The solution process is the same as 3.1. computes the Hessian matrix and obtain \( \beta < \frac{5}{7} \) so we assume \( \beta < \frac{5}{7} \) and the results are shown in Table 2.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Equilibrium under the collusion of duopoly retailers (( cn ) means collusion)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>wholesale price (( w_m^c ))</td>
</tr>
<tr>
<td></td>
<td>assistance per unit CS effort (( x_b^c = -\frac{\alpha_1+\alpha_2}{4(\beta-1)} ))</td>
</tr>
<tr>
<td></td>
<td>bonus per unit CS effort (( \eta^c = 0 ))</td>
</tr>
<tr>
<td>Stage 2</td>
<td>retailer 1’s selling price (( p_1^c = \frac{Iw_m^c - 2F(x_b^c + \eta^c) - 2G}{Y} ))</td>
</tr>
<tr>
<td></td>
<td>retailer 1’s CS effort (( b_1^c = \frac{Fw_m^c - 2F(x_b^c + \eta^c) - G}{Y} ))</td>
</tr>
<tr>
<td></td>
<td>retailer 2’s selling price (( p_2^c = \frac{Iw_m^c - 2F(x_b^c + \eta^c) - 2H}{Y} ))</td>
</tr>
<tr>
<td></td>
<td>retailer 2’s CS effort (( b_2^c = \frac{Fw_m^c - 2F(x_b^c + \eta^c) - H}{Y} ))</td>
</tr>
</tbody>
</table>

Note: \( 0 \leq \beta < \frac{5}{7} \). \( Y = 16\beta^2 - 9, F = -4\beta^2 + \beta + 3, G = 4\beta \alpha_2 + 3\alpha_1, H = 4\beta \alpha_1 + 3\alpha_2, I = 8\beta^2 - 2\beta - 3, J = 4\beta + 3. \)
Proposition 2:
(1) The manufacturer only provides CS assistance in the CS incentive plan.
(2) When the degree of substitutability β between retailers increase, the manufacturer should increase CS assistance in the CS incentive plan.

Prove. In order to prove Proposition 2 (2), solving the partial derivative of \( x_p^m \) with respect to \( β \) in Table 2 is needed.

\[
\frac{\partial x_p^m}{\partial β} = \frac{2β(α_1 + α_2)}{(2β - 20)^2} > 0 \tag{7}
\]

Due to \( \frac{\partial x_p^m}{\partial β} > 0 \), \( x_p \) under collusion game is increased with \( β \).

Proposition 2 (1) indicates that when retailers adopt a collusion approach, manufacturers only provide CS assistance to retailers. The results of Huang et al. [11] showed that retailers' collusion behaviour will increase their selling prices, thus causing the manufacturer to encounter lower profits, and the retailer's collusion behaviour will cause the manufacturer to charge a lower wholesale price. Therefore, the manufacturer will only provide CS assistance to retailers and cancel the CSI bonus.

Proposition 2 (2) further indicates that when \( β \) increase, the manufacturer should give retailers more CSI bonuses and CS assistance.

3.3 Impact of the strategy

This section discusses the impact of the Cournot and collusion behaviour of duopoly retailers on each of the equilibrium solutions in Table 1 and Table 2, and analyse the preferences of manufacturers and retailers. The following discussion is in \( β < \frac{5}{7} \) the case was carried out, in which the superscript \( ct \) represents Cournot, and the superscript \( cn \) represents collusion.

Proposition 3: \( w_m^{cn} < w_m^{ct} \).

Prove. \( w_m^{cn} \) with \( w_m^{ct} \) make a difference

\[
w_m^{cn} - w_m^{ct} = -\beta(α_1 + α_2) > 0 \tag{8}
\]

The difference between the two is less than 0, so the proposition is proved.

Proposition 3 indicates that the retailer's collusion behaviour will cause the manufacturer to charge lower wholesale price. Consistent with the conclusions in literature [11].

Proposition 4: \( π_m^{cn} < π_m^{ct} \).

Prove. \( π_m^{cn} \) with \( π_m^{ct} \) make a difference

\[
π_m^{cn} - π_m^{ct} = \begin{cases} 
\frac{(588β^4 - 16712β^2 + 2448β + 1045)(α_1 + α_2)^2}{8(6β^2 - 17β + 10)(4β - 3)(7β - 5)^2} < 0 & \beta > \frac{1}{4} \\
\frac{(2056β^5 - 49586β^4 + 195380β^3 - 43905β^2 - 7800)(α_1 + α_2)^2}{8(2β - 3)(4β - 3)(7β - 5)^2(6β - 5)^2} < 0 & \beta ≤ \frac{1}{4}
\end{cases} \tag{9}
\]

The difference between the two is less than 0, so the proposition is proved.

Proposition 4 indicates that the retailer's collusion behaviour will cause the manufacturer to suffer lower profits, because the collusion behaviour of retailers will increase selling prices and reduce sales.

Proposition 5: \( p_1^{cn} > p_1^{ct} \), if \( β > 0.25 \); \( p_2^{cn} > p_2^{ct} \), if \( β ≤ 0.25 \) and \( α_1 ≤ α_2 \); \( p_2^{cn} > p_2^{ct} \), if \( β ≤ 0.25 \) and \( α_1 ≥ α_2 \); \( b_1^{cn} > b_1^{ct} \).

Prove. First \( p_1^{cn} \) with \( p_1^{ct} \) make a difference

\[
p_1^{cn} - p_1^{ct} = \begin{cases} 
\frac{α_1 + α_2}{2(4β - 3)} - \frac{7α_1 + 7α_2}{4(7β - 5)} - \frac{α_1 + α_2}{4(β - 1)} + \frac{3α_1 + 3α_2}{8β - 5} + \frac{α_1 + α_2}{4β - 3} > 0 & \beta > \frac{1}{4} \\
\frac{α_1 + α_2}{2(4β - 3)} - \frac{7α_1 + 7α_2}{4(7β - 5)} + \frac{2α_1 - α_2}{8β - 5} > 0 & β ≤ \frac{1}{4}
\end{cases} \tag{10}
\]
When $\beta \leq \frac{1}{4}$, \( \frac{a_1+a_2}{2(4\beta-3)} - \frac{7a_1+7a_2}{4(7\beta-5)} - \frac{a_1+a_2}{4(\beta-1)} + \frac{9a_1+9a_2}{8(6\beta-5)} + \frac{5a_1+5a_2}{8(2\beta-3)} > 0 \); So if $\frac{-a_1-a_2}{2\beta+3} + \frac{a_1-a_2}{4\beta+3} = -\frac{2(a_1-a_2)\beta}{8\beta^2+18\beta+9} \geq 0$, then $p_{ct}^m - p_{ct}^r$ is greater than 0. If $\frac{-a_1-a_2}{2\beta+3} + \frac{a_1-a_2}{4\beta+3} \geq 0$, then $\alpha_1 \geq \alpha_2$.

Similarly, $p_{2ct}^m$ with $p_{2ct}^r$ make a difference

$$p_{2ct}^m - p_{2ct}^r = \left( \frac{a_1+a_2}{2(4\beta-3)} - \frac{7a_1+7a_2}{4(7\beta-5)} - \frac{a_1+a_2}{4(\beta-1)} + \frac{9a_1+9a_2}{8(6\beta-5)} + \frac{5a_1+5a_2}{8(2\beta-3)} > 0 \right) \beta \geq \frac{3}{4}$$

$$p_{2ct}^m - p_{2ct}^r = \left( \frac{a_1+a_2}{2(4\beta-3)} - \frac{7a_1+7a_2}{4(7\beta-5)} - \frac{a_1+a_2}{4(\beta-1)} + \frac{9a_1+9a_2}{8(6\beta-5)} + \frac{5a_1+5a_2}{8(2\beta-3)} > 0 \right) \beta \geq \frac{3}{4}$$

The difference between the above two is greater than 0, so the proposition is proved.

Proposition 5 indicates that the retailer's collusion behaviour will cause them to charge higher selling prices and make greater CS efforts. When retailers raise their selling prices, sales will decrease. At this time, in order to avoid excessive sales reduction, retailers will make greater effort to increase CS to increase sales. In other words, when retailers choose collusion behaviour, they will confront the reduction in sales due to rising selling prices by increasing CS efforts.

Proposition 6: $\pi_{ct}^m > \pi_{ct}^r$.

Prove. Subtract $\pi_{ct}^m$ from $\pi_{ct}^r$, and $\pi_{ct}^m$ from $\pi_{ct}^r$ separately.

$$\pi_{ct}^m - \pi_{ct}^r = \left( \frac{-3(a_1-a_2)^2 + -3(a_1+a_2)^2 + \frac{(-a_1+a_2)^2}{2(2\beta+3)} + \frac{(a_1+a_2)^2}{2(2\beta+3)} + \frac{a_1-a_2}{2(4\beta+3)} + \frac{a_1-a_2}{2(4\beta+3)}}{2(2\beta+3)} \right) \beta \geq \frac{1}{4}$$

$$\pi_{ct}^m - \pi_{ct}^r = \left( \frac{-3(a_1-a_2)^2 + -3(a_1+a_2)^2 + \frac{(-a_1+a_2)^2}{2(2\beta+3)} + \frac{(a_1+a_2)^2}{2(2\beta+3)} + \frac{a_1-a_2}{2(4\beta+3)} + \frac{a_1-a_2}{2(4\beta+3)}}{2(2\beta+3)} \right) \beta \geq \frac{1}{4}$$

The difference between the above two is greater than 0, so the proposition is proved.

Proposition 6 indicates that regardless of the initial reputation of the two retailers and the degree of substitutability between the two retailers, they will choose collusion behaviour because collusion result will in higher profits.

Besides discussing the respective profits of manufacturers and retailers, the profits of the entire supply chain should also be taken into account. The total profit of the supply chain $\pi_A^f$ is:

$$\pi_A^f = \pi_m^f + \pi_r^f + \pi_r^f \quad (f = ct or cn)$$

Advances in Production Engineering & Management 15(3) 2020
Proposition 7: \( \pi_A^{cn} > \pi_A^{ct} \), If \( \beta \geq 0.20 \); \( \pi_A^{cn} < \pi_A^{ct} \), If \( \beta \leq 0.15 \).

Prove. \( \pi_A^{cn} \) and \( \pi_A^{ct} \) make a difference

\[
\pi_A^{cn} - \pi_A^{ct} = \begin{cases} 
\frac{(a_1+a_2)^2}{8(4\beta-3)} + \frac{(a_1-a_2)^2}{2(4\beta+3)} + \frac{(a_1+a_2)^2}{8(7\beta-5)} + \frac{7(a_1-a_2)^2}{8(7\beta-5)} - \frac{3(a_1-a_2)^2}{2(2\beta+3)^2} - \frac{3(a_1+a_2)^2}{8(6\beta-5)^2} + \beta > \frac{1}{4} \\
\frac{3(a_1+a_2)^2}{8(6\beta-5)} - \frac{(a_1+a_2)^2}{8(\beta-2)} - \frac{(a_1-a_2)^2}{8(\beta-2)} - \frac{7(a_1-a_2)^2}{8(7\beta-5)} - \frac{3(a_1-a_2)^2}{2(2\beta+3)^2} + \frac{19(a_1+a_2)^2}{128(6\beta-5)^2} & \beta \leq \frac{1}{4} 
\end{cases} 
\] (17)

When \( \beta > \frac{1}{4} \), \( \pi_A^{cn} - \pi_A^{ct} > 0 \); But when \( \beta \leq \frac{1}{4} \), \( \pi_A^{cn} - \pi_A^{ct} \) there are both greater than 0 and less than 0, so discussions were made at \( \beta = 0.05, 0.1, 0.15, 0.20, 0.25 \). When \( \beta = 0.05, 0.1, 0.15 \), \( \pi_A^{cn} - \pi_A^{ct} < 0 \); When \( \beta = 0.20, 0.25 \), \( \pi_A^{cn} - \pi_A^{ct} > 0 \), so the proposition is proved.

Proposition 7 indicates that when \( \beta \) is large, collusion will make the entire supply chain getting more profit. When \( \beta \) is small, Cournot will make the entire supply chain getting more profit. Because there is an alternative relationship between the needs of the two retailers. When this alternative relationship is stronger (\( \beta \) larger), it will have a greater impact on the respective needs of the two retailers. The collusion between the two parties will reduce the impact on demand, thus making the entire supply chain more profits. When the substitution relationship is weaker (\( \beta \) smaller), the impact on the respective needs of the two retailers is smaller. Therefore, the two parties are unwilling to collusion, they all hope to maximize their profits through competition, thus making the entire supply chain more profits.

3.4 The impact of substitutability \( \beta \) on decision results

This section discusses the impact of substitutability \( \beta \) on optimal wholesale price, selling price, profit, CS effort, and CS incentives. Let \( \alpha_1 = 100, \alpha_2 = 90 \), the calculation results are shown in the figures Figs. 1 to 6. Fig. 1 and Fig. 2 show that the wholesale price and profit of the manufacturer under two games increase with the increase of \( \beta \). When \( \beta = 0 \), the wholesale price and profit under the two games are the same. With the increase of \( \beta \), the manufacturer's wholesale price difference between the two games will also increase, while the manufacturer's profit difference between the two games will increase first and then decrease. Therefore, the cross-price effect of downstream retailers will have a significant impact on the manufacturer's wholesale price and profit. When \( \beta > 0 \), the wholesale price and profit of the manufacturer under the Cournot game of downstream retailers are larger, which is consistent with Proposition 3 and Proposition 4.
Manufacturer's customer satisfaction incentive plan for duopoly retailers with Cournot or collusion games

Fig. 3, Fig. 4 and Fig. 5 show that the optimal selling price, CS effort and profit of retailer $i$ under two games increase with the increase of $\beta$. When $\beta = 0$, the selling price, CS effort and profit of the same retailer under the two games are the same. With the increase of $\beta$, the difference of selling price, CS effort and profit of each retailer between the two games will also increase. When $\beta > 0$, the selling price, CS effort and profit of the retailer $i$ under the collusion game of downstream retailers are larger. This is because when downstream retailers play collusion game, they will collude to increase their own interests and squeeze the interests of the manufacturer. This is why the collusion of downstream retailers is illegal in some countries.

We can see from Fig. 6 that the manufacturer’s CS assistance per unit will increase with the increase of $\beta$ under the two games. When $\beta = 0$, the two are equal, when $\beta > 0$, the CS assistance per unit is larger than in the collusion game of retailers. When $0 < \beta \leq 0.25$, the manufacturer’s CSI bonus per unit was 0. When $\beta > 0.25$, the CSI bonus with the Cournot game will still be 0, while the CSI bonus in collusion game will increase with the increase of $\beta$, and the increase rate is faster, which will gradually be greater than the CS assistance in the Cournot game.
4. The impact of manufacturer CS incentives

The CS incentive is provided by the manufacturer, therefore, from the perspective of the manufacturer, we study the impact of the CS incentive on the manufacturer. First, we must find the manufacturer’s decision when the CS incentive is not provided, and then compare them with the decision in section 3. Here, we use (−) to represent the results in the case of no CS incentives.

Table 3 Equilibrium under duopoly retailers’ two games without CS incentive

<table>
<thead>
<tr>
<th>Stage</th>
<th>Optimal value</th>
<th>Cournot (j = ct)</th>
<th>Collusion (j = cn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 1</td>
<td>( w_m )</td>
<td>( \frac{a_1 + a_2}{4} )</td>
<td>( \frac{a_1 + a_2}{4} )</td>
</tr>
<tr>
<td>Stage 2</td>
<td>( p_1 ), ( b_1 ), ( p_2 ), ( b_2 )</td>
<td>( \frac{(2\beta + 3)w_m^l + 4\beta a_2 + 6a_1}{4\beta^2 - 9} )</td>
<td>( \frac{(8\beta^2 - 2\beta - 3)w_m^l - 8\beta a_2 - 6a_1}{4\beta^2 - 9} )</td>
</tr>
</tbody>
</table>

4.1 Game analysis when manufacturers do not provide CS incentives

Assuming that the manufacturer will not provide CS incentives to retailers, let's analyse this two-stage decision-making problem, in which the demand function is same as Eq. 1. Retailer 1's profit function \( \pi_i = (p_i - w_m)D_i - b_i^2 \) (18)

The manufacturer's profit function \( \pi_m \) is:

\[ \pi_m = w_m(D_1 + D_2) \] (19)

Similarly, we use the backward induction method to obtain the equilibrium without any CS incentive plan, and the results are shown in Table 3.

Bringing the equilibrium in Table 3 into Eq. 19, the manufacturer’s profits in the two models without any CS incentive plan respectively are:

\[ \pi_m^{ct} = \frac{(a_1 + a_2)^2}{8\beta^2 - 20\beta + 12} \] (20)

\[ \pi_m^{cn} = -\frac{(a_1 + a_2)^2}{16\beta - 12} \] (21)

4.2 The impact of CS incentives on manufacturer’s profit and wholesale price

We compare the wholesale price and profit of the manufacturer in the two cases with or without CS incentives.

Proposition 8: The manufacturer's CS incentives may or may not increase its wholesale price, it depends on which game the duopoly retailer adopts, but it will certainly obtain higher profits.

\[ w_{m}^{ct} - w_{m}^{cn} = \frac{(a_1 + a_2)\beta}{24\beta^2 - 4\beta + 20} > 0 \] (22)

\[ w_{m}^{cn} - w_{m}^{ct} = 0 \] (23)

\[ \frac{\pi_m^{ct} - \pi_m^{cn}}{\pi_m^{ct}} = \begin{cases} \frac{2\beta^2 - \beta + 1}{12\beta^2 - 34\beta + 20} & \beta > \frac{1}{4} \\ \frac{-2\beta^2 + 1}{(6\beta - 5)^2} & \beta \leq \frac{1}{4} \end{cases} \] (24)

\[ \frac{\pi_m^{cn} - \pi_m^{ct}}{\pi_m^{cn}} = \frac{3 - 2\beta}{2(7\beta - 5)^2} > 0 \] (25)

From Eq. 22 and Eq. 23, we can see that when the retailers adopt a Cournot game, the manu-
manufacturer’s CS incentive plan will increase its wholesale price; when the retailers adopt a collusion game, its wholesale price remains unchanged. Because Eq. 24 is greater than 0, \( \pi_{ct,m}^c > 0 \), so \( \pi_{ct,m}^c - \pi_{ct,m}^c = 0 \); Because Eq. 25 is greater than 0, \( \pi_{cn,m}^c > 0 \), so \( \pi_{cn,m}^c - \pi_{cn,m}^c > 0 \). Therefore, no matter which game the retailers adopt, the manufacturer’s CS incentive plan will make it obtain higher profits.

4.3 The impact of CS incentives on consumer’s demand

Besides whole sale price, the demand is also an important issue for the future success of the manufacturer. Therefore, in this section, we will discuss the impact of CS incentives on consumer’s demand. The consumer’s demand function is:

\[
D_m = D_1 + D_2 = \alpha_1 + \alpha_2 - p_1 - p_2 + b_1 + b_2 + \beta(p_1 + p_2) \tag{26}
\]

As shown in Fig. 7, no matter what game the two retailers adopt, the manufacturer’s CS incentive plan will increase their demand, and its demand will increase with the increase of \( \beta \). In the case that the manufacturer provides CS incentives, when \( \beta = 0 \), the demand in the two games is equal. With the increase of \( \beta \), first the demand in the Cournot game is larger, and then the demand in the collusion game went up quickly and exceed that in the Cournot game. In the case that the manufacturer does not provide CS incentives, when \( \beta = 0 \), the demand in the two games is equal. Similarly, with the increase of \( \beta \), the demand in the Cournot game is larger at the beginning, but then the demand in the collusion game becomes larger than that in the Cournot game.

5. Conclusion

This paper examined a two-echelon supply chain system consisting of a manufacturer and duopoly retailers. The manufacturer offers retailers two types of CS incentives to enable retailers to improve CS. First, the paper establishes two models to study the best CS incentive plan for duopoly retailers in the context of Cournot and collusion. Then, compare and analyse the equilibrium results of the two models. Finally, we study the impact of the CS incentive plan on the manufacturer. The results showed that:

- When the two retailers take collusion behaviour, the manufacturer only provides CS assistance to the retailers but no CSI bonus.
- In the Cournot game condition if the wholesale price is greater than the threshold, the manufacturer provides CSI bonus and CS assistance to the retailers; if the wholesale price is less than the threshold, the manufacturer only provides CS assistance to the retailers.
- In both models, when the degree of substitutability between retailers increases, the manufacturer should always increase CS assistance in the CS incentive plan. In the Cournot
model, when the degree of substitutability between retailers increases, the manufacturer should increase CSI bonuses in the CS incentive plan.

- Retailers always like collusion behaviour, because collusion behaviour will cause manufacturers to lower wholesale prices, which will enable retailers to increase selling prices and make greater CS efforts, thus enabling retailers to higher profit. Manufacturers always like Cournot behaviour, because retailers’ Cournot behaviour will make them more profitable.
- The decision results of the best wholesale price, profit, selling price and CS effort will increase as the degree of substitution between retailers increases.
- The manufacturer’s CS incentives may or may not increase its wholesale price, which mainly depends on what kind of game the retailers adopt, but it will certainly obtain more demand and higher profits.

According to the above analysis and proposition, the manufacturer should fully consider the marketing mix decisions between retailers when formulating CS incentive plans, and try to cooperate with retailers with a high degree of substitution. Although the model proposed in this paper is more in line with the enterprise production management practice, there are still some limitations. In the demand function, we assumed that the sensitivity of the CS effort is 1, but it may not always be the case in real life. Therefore, the demand function needs to be improved. The downstream market is generally composed of multiple retailers, and this study only considers two retailers. In addition, this paper only considers the Cournot and collusion game of duopoly retailers, and future research can consider Stackelberg game.

Acknowledgement

This work is supported by National Natural Science Foundation of China (No. 71704151), Research funding of Hebei Key Research Institute of Humanities and Social Sciences at Universities (JJ1907).

References


