Supply chain coordination based on the probability optimization of target profit

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ABSTRACT

Supply chain management decision-making study mainly based on the expected utility theory and most of the studies are obtaining the average values in the statistical sense. For Supply Chain (SC) decision-making individuals the statistical-based optimal profitability brings decision conflicts in the particular market within a specific period. Moreover, the small and medium outsourcing participants face unexpected outcomes which are the main cause of SCs disruption. This study proposes a contractual coordination model that maximizes the probability of a pre-determined Profit Target (PT). The purpose of this paper is to reduce the influence of demand uncertainty with the high risk of unexpected outcomes. We constructed the Revenue Sharing (RS) and buyback contract models within the SC participants’ PT conditions and then discussed the SC overall performance. We simulated and analyzed the coordination conditions and the decision-making preferences of SC participants under the two contracts. From the comparison, under the PT strategy, the retailer is more willing to adopt the RS contract rather than the buyback contract. But the SC upstream supplier’s contract selection decision depends on the specific contract parameters. Finally, numerical results indicated the contract selection decisions with the given PT of both SC participants.

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1. Introduction

Fast-growing market competition is increasing the SC market demand uncertainty which is one of the main causes of SCs disruption. Moreover, the healthcare industry and short life-cycle products’ SCs face unpredictable and unexpected market returns caused by the combination of industry competitiveness and perishability. The management of SC with the Profit Target based on decision-makers’ incentivization has multiple advantages in today’s marketplace. The stability of a SC under the PT is one of the main advantageous motivations. Today’s healthcare industry requires a new strategy to deal with high risks and ensure their market’s satisfaction stability. Moreover, the SCs engaged in socially responsible activities and jointly intend to exhibit Corporate Social Responsibility (CSR) with a pre-determined PT may be a compromise solution. The CSR participant’s profit may be negative and to keep their stability, the management of SC requires a PT level. However, the SCs with CSR solutions are under pressure of social and environmental issues [1]. The channels with a specific PT have unique profitability influence and the SCs who fail to
maximize their adoption of a new strategy will be disrupted by the market uncertainty. Additionally, a SC under the PT orientation can also ensure the profitability of chain participants within the demand uncertainty. PT might help the SC participants reduce risk and loss with a specified level of profit. PT is part of management strategies that SC participants use to manage the risks. "A profit target is a pre-determined point at which the SC participants can initiate conditional orders in a predictable and specified level as well as maximum loss constraints".

The existing research in SC are mainly based on the expected utility theory, with the utility maximization as the decision-making goal. The expected utility theory is a statistical-based mean concept, which brings unexpected outcomes for SC decision-makers. Therefore, decision-making studies based on PT are promising new approaches. In the SC decision-making study with the PT, the probability of realizing a PT is maximized as the decision-making goal. Compared with the expected utility theory, the SC decision-maker can cooperate more intuitively, and it can effectively reduce the loss caused by market fluctuations.

The newsboy model is a single item inventory controlling problem within the single-period stochastic demand, where demand uncertainty is the main industry issue under inventory, pricing, and overall operational management studies [2]. Khouja investigated and reviewed the different single-period problem-based extensions [3]. The research on supply chain can be summarized into the following categories. The first is the research on the supply chain structure, which can be divided into two-level supply chain [4], three-level supply chain [5], and two-channel supply chain [6]. On this basis, there are also literature studies on the optimization of channel structure [7]. The second is to study the uncertainty risk in the supply chain [8]. Then there is the research on information asymmetry [9]. Lee et al. [10] defined the content of information sharing in the supply chain and constructed the basic model of information sharing in the supply chain [11]. Finally, the relevant research is based on the preference of decision makers. The common preferences are risk aversion [12], waste aversion [13], fairness concern [14], test aversion [15] and green preference [16].

The single-period problem or newsboy model-based first PT investigation with a two-product SC extension was proposed in [17]. Their study constructed a two-product SC model with a newsboy problem and investigated the effects of SC participants’ decision-making on achieving the PT. The newsboy problem-based research with pricing was proposed in [18]. Yang et al. research considered both profit and revenue targets under the single-period problem, the study discussed the effects of profit and revenue targets on the expected profits and the probability of achieving PT or revenue target respectively [19].

Available literature mainly considered the single decision maker’s behavior in a SC with the PT and the influence of various newsboy products on a SC. None of the literature considers the joint strategical decision-making of the SC participants in-between an upstream and a downstream chain participant with their PT. However, most of the research with the newsboy problem is concerned with profit maximization. Additionally, there is a lack of studies simultaneously engaged with the stability of a SC under the PT and profit maximization.

Contractual coordination provides a viable alternative as a mechanism for coordination of the SC and proves to be an interesting research direction. Various decentralized SC processes have been coordinated aiming to improve the functionality. The concept of contractual coordination in improving the SC processes have been widely investigated. Researchers and practitioners used incentivization to motivate and coordinate their SC performance [20]. The contractual coordination mechanisms are important to have the decentralized SC's decision-makers pursue channel coordination. The existing SC contractual coordination mechanism with the newsboy problem is proposed by Arrow and Harris [21]. The most common applied contractual coordination mechanisms include wholesale price contracts [21], RS contracts [22], buyback contracts [23], and quantity flexibility contracts [24]. Considering contractual coordination under the newsboy problem, the buyback contract is incentivizing to increase an order quantity by sharing the inventory risk of downstream in a SC. The RS contract with the newsvendor problem plays an important role in the coordination of SC and most of the studies focused on newsvendor given with exogenous retailing price. Several extensions to the earlier contract model with newsboy have been developed [25]. Upstream supplier maximizes the profit of whole decentralized SC as the coordinator, in this
case, the RS contract model can efficiently coordinate SC members’ performance. The position guarantees a contractual coordination improvement, whereas the sharing parameter determines SC total profit distribution in between members [26]. The literature on SC contract mainly focuses on profit allocation. No research investigates the PT orientated SC under the contractual coordination theory to reduce the influence of demand uncertainty.

This study constructs a contractual coordination model based on the PT oriented chain participants’ incentivization. We analyzed the RS and buyback contract models with a SC participant’s PT and then discussed the contract selection within SC participants’ PT. Then, we characterized the decision-makers’ optimal profit realization and the contract selection conditions, to reduce the influence of demand uncertainty with the risk of unexpected loss. This paper expands the existing research scenarios of supply chain contracts, which is closer to the actual production situation of the market for decision makers, and also provides a reference for the design of diverse contracts.

The rest of this paper is organized as follows. Section 2 provides the basic model description, establishment, and assumptions. Section 3 analyses of the RS contract with a specific PT and coordination conditions of SC. Section 4 provides the analyses under the buyback contract with SC PT and coordination conditions. Section 5 presents the numerical analysis of contract selection with the given PT. Section 6 summarizes and concludes the study results.

2. The basic model, description, and assumptions

In this section, we construct a basic contract model with a PT similar to Yang et al. [19]. Consider a two-stage SC consisting of one upstream supplier and one downstream retailer. The market demand during the single-season is stochastic. To avoid an unexpected random demand outcome that will cause chain disruption, the SC participants will set a PT. The PT is pre-determined and expected to be achieved in advance. The probability of achieving a PT is SC participants’ decision-making variable as the self-interested on the stability and adoption in a SC.

To facilitate this model, this paper has the following assumptions:

**Assumption 1:** The SC upstream and downstream participants are rational decision-makers.

**Assumption 2:** The SC participants will determine the optimal order quantity and the maximum probability of achieving the PT based on the pre-determined target point.

**Assumption 3:** The SC shortage loss is not considered.

**Assumption 4:** The SC downstream retailer has only one opportunity to decide the ordering variable. As the classical newsboy problem, there is only one chance to order at the beginning of a single season.

We consider the SC contract parameters with a unit wholesale price $w$, a unit selling price $p$, and a unit salvage value $v$. The market demand is $x$ and we denote $F(x)$ as the distribution function of $x$ with its density $f(x)$.

**Expected profit of downstream retailer**

For the comparison, we first characterized a benchmark case with the retailer’s profit function. Within the classical newsboy problem, SC downstream retailer’s order quantity $q$ is under the conditions of:

$$\begin{align} r &= \begin{cases} px + v(q - x) - wq & x < q \\ pq - wq & x \geq q \end{cases} \end{align}$$

Then, the expected profit of a SC downstream retailer under the newsboy problem is as follow:

$$E_{\pi_r} = \int_0^q (px + v(q - x) - wq)f(x)dx + \int_q^{+\infty} (pq - wq)f(x)dx$$

For the retailer’s profit analysis, where the downstream retailer set a PT, we donate the constraint conditions of order quantity. When $x < q$ the retailer’s profit increases with the increase of market demand; else when $x \geq q$ the retailer’s profit does not change with the change of market demand and the maximum profit can be reached when $x = q$. If the retailer has a PT $t_r$ to
achieve, it cannot exceed the maximum profit, as \( t_r \leq pq - wq \) and \( q \geq \frac{t_r}{p - w} \). Thus, if \( q < \frac{t_r}{p - w} \), then \( t_r > pq - wq \), the probability of achieving the PT is 0. When \( q \geq \frac{t_r}{p - w} \) in order to achieve the PT, the condition needs to satisfy \( px + v(q - x) - wq \geq t_r \). Hence, we can get \( x \geq \frac{(w-v)q+t_r}{(p-v)} \) and the probability of \( x \geq \frac{(w-v)q+t_r}{(p-v)} \) is \( 1 - F\left(\frac{(w-v)q+t_r}{(p-v)}\right) \). Therefore, the probability of the SC retailer’s PT is:

\[
P(t_r) = \begin{cases} 0 & q < \frac{t_r}{p - w} \\ 1 - F\left(\frac{(w-v)q+t_r}{(p-v)}\right) & q \geq \frac{t_r}{p - w} \end{cases}
\]

When \( q < \frac{t_r}{p - w} \), the probability of the SC retailer’s PT is 0; when \( q \geq \frac{t_r}{p - w} \), the probability of the retailer achieving the PT increases with the increase of \( q \). The optimal order quantity of the retailer can be obtained as \( \frac{t_r}{p - w} \) where the probability of the retailer’s PT is \( 1 - F\left(\frac{t_r}{p - w}\right) \).

**Definition 1**: In the SC downstream retailer’s decision on maximizing the probability, he first sets a PT \( t_r \) based on his actual situation. In order to achieve a PT, the retailer’s optimal order quantity is \( \frac{t_r}{p - w} \) and the probability of achieving the PT is maximum \( 1 - F\left(\frac{t_r}{p - w}\right) \).

**Expected profit of upstream supplier**

SC upstream supplier has a unit production cost \( c \) and the total profit of the supplier can be expressed as follow:

\[
\prod_s = wq - cq
\]

Where the expected profit of the SC upstream supplier under the newsboy problem is as follow:

\[
E_{\pi_s} = \int_0^{+\infty} (wq - cq)f(x)dx
\]

For the SC upstream supplier, who does not face the market demand directly, the profit will not change with the change of market demand. Therefore, the profit that the supplier can achieve is \( wq - cq \), and the probability of achieving her PT is 100%.

From the basic model analysis, we can get the following three aspects of the problem:

- Comparing to the SC downstream retailer conditions, the upstream supplier does not bear the market risk, and her profit will not change with the change of market demand. The supplier only incentivized to get the downstream retailer’s order as much as possible, so that she can achieve higher profit;
- When the SC retailer determines the PT, it can increase the probability of achieving the PT by adjusting his order quantity. There is an optimal order quantity under each specific PT for retailer, but the SC downstream’s order quantity is not optimal from the point of supplier and the overall SC.
- For the SC retailer and supplier, the level of PT that can be achieved by both participants are less under the decentralized decision-making control.

Thus, the research of PT oriented SC contracts analyzes both participants’ decision-making in order to find the overall SC coordination conditions.

**Definition 2**: When the SC participants are making an uniform decision in order to maximize their probability of achieving PT with an optimal order quantity, we define this condition as the SC coordination.
3. Revenue-sharing contract with a specific profit target

In this section, we construct the RS contract with PT oriented SC retailer and supplier, then we consider the coordination conditions under the RS contract from the perspective of SC participants’ PT. The downstream retailer is a SC leader and the upstream supplier is a follower.

Based on the Definition 1, the events of the decision-making process under the RS contract with SC participants’ PT and based on the Definition 2, the analyzes of the coordination conditions under the RS contract with SC participants’ PT are as follows:

1) SC downstream retailer and upstream supplier set their PT.
2) SC upstream supplier is committed to providing for the retailer a lower wholesale price, while the downstream retailer is committed to returning a certain percentage of sales revenue.
3) SC retailer and supplier will determine their PT and set the optimal order quantity.
4) According to Definition 2, obtaining the PT based RS contract coordination conditions.
5) Analyzing the RS contract conditions for the improvement of the overall SC profitability.

3.1 Analysis of the retailer’s decision

Under the RS contract, the supplier charges wholesale price \( w_\varphi \) per unit product while the downstream retailer is committed to returning a certain percentage \( (1 - \varphi) \) of sales revenue to the supplier for making up for the supplier’s profit loss due to the lower wholesale price to the retailer. Thus, the total profit of the downstream retailer can be expressed as:

\[
\prod r = \begin{cases} 
\varphi(px + \nu(q - x)) - w_\varphi q & x < q \\
\varphi p q - w_\varphi q & x \geq q 
\end{cases} \quad (5)
\]

Where \( \varphi \) assumed to be in the range of \( 0 < \varphi < 1 \), and the expected profit of the downstream retailer under the newsboy problem is as follow:

\[
E_{\pi_r} = \int_0^q \left( \varphi (px + \nu(q - x)) - w_\varphi q \right) f(x)dx + \int_q^{+\infty} (\varphi p q - w_\varphi q) f(x)dx \quad (6)
\]

For the retailer’s PT analysis, when \( t_r > (\varphi p - w_\varphi)q \), we have \( P(t_r) = 0 \), that is, when \( q < \frac{t_r}{\varphi p - w_\varphi} p(t_r) = 0 \); when \( t_r = (\varphi p - w_\varphi)q \), which is \( q = \frac{t_r}{\varphi p - w_\varphi} \), we have \( P(t_r) = 1 - F(q) \); and when \( t_r \leq (\varphi p - w_\varphi)q \), or equivalently \( q > \frac{t_r}{\varphi p - w_\varphi} \), we have \( P(t_r) = 1 - F \left( \frac{(w_\varphi - \varphi q)q + t_r}{\varphi p - \varphi} \right) \). Therefore, the probability of the retailer’s PT under RS contracts is:

\[
P(t_r) = \begin{cases} 
0 & q < \frac{t_r}{\varphi p - w_\varphi} \\
1 - F \left( \frac{(w_\varphi - \varphi q)q + t_r}{\varphi p - \varphi} \right) & q \geq \frac{t_r}{\varphi p - w_\varphi} 
\end{cases}
\]

**Theorem 1:** Under the RS contract, the optimal order quantity of the SC retailer is \( t_r \frac{\varphi p - w_\varphi}{w_\varphi} \) – and the probability of achieving PT is \( 1 - F \left( \frac{t_r}{\varphi p - w_\varphi} \right) \).

**Proof of Theorem 1:** In the case where \( q < \frac{t_r}{\varphi p - w_\varphi} \) the probability of achieving retailer’s PT is 0. In the case where \( q \geq \frac{t_r}{\varphi p - w_\varphi} \), \( w_\varphi - \varphi q > 0 \) we have \( \frac{(w_\varphi - \varphi q)q + t_r}{\varphi p - \varphi} \) which is increases with the increase of \( q \). \( 1 - F \left( \frac{(w_\varphi - \varphi q)q + t_r}{\varphi p - \varphi} \right) \) decreases with the increase of order quantity \( q \). \( F(x) \) is an increasing function. Therefore, the retailer’s optimal order quantity is \( \frac{t_r}{\varphi p - w_\varphi} \) and the probability of achieving PT is \( 1 - F \left( \frac{t_r}{\varphi p - w_\varphi} \right) \).

**Corollary 1:** Under RS contract condition, \( \varphi > \frac{p - w + w_\varphi}{p} (w > w_\varphi) \), retailers can increase the probability of achieving PT by increasing the RS contract coefficient \( \varphi \).
Proof: To increase the probability of SC retailer’s PT achievement, the conditions of $1 - F\left(\frac{t_r}{\phi p - w}\right) > 1 - F\left(\frac{t_r}{p - w}\right)$ must be satisfied. It is equivalent to $\frac{t_r}{p - w} > \frac{t_r}{\phi p - w}$, thus we have $\phi > \frac{p - w + w\phi}{p} (w > w\phi)$. By taking the derivative of $1 - F\left(\frac{t_r}{\phi p - w}\right)$ with respect to $\phi$, we have $f \left(\frac{t_r}{\phi p - w}\right) * \frac{p t_r}{(-w + p\phi)^2} > 0$ for any $\phi$. Therefore, the probability of achieving PT $1 - F\left(\frac{t_r}{\phi p - w}\right)$ increases with the increase of RS contract coefficient $\phi$.

Fig. 1 The probability of achieving the PT: different sharing coefficient vs. no contract

Fig. 1 illustrates a comparison of the SC retailer’s probability of achieving the PT under different contract parameters and no contract case, where $P_A$ represents the probability of achieving the PT with no contract case; $P_B$ represents the probability of achieving a PT under the condition where the RS contract coefficient satisfied $\phi < \frac{p - w + w\phi}{p}$, $P_C$ represents the probability of achieving a PT under where the RS contract coefficient is larger, i.e., $\phi > \frac{p - w + w\phi}{p}$. From Fig. 1 we can get the conditions of $P_C > P_A > P_B$.

Fig. 2 illustrates a comparison of retailer’s probability of achieving PT under different targets and no contract case respectively, where $t_{01} = t_{r1} < t_{02} = t_{r2}$. Consequently, Figs. 1 and 2 indicate that if $\phi < \frac{p - w + w\phi}{p}$ retailer’s probability of achieving its PT will be significantly reduced. On the contrary, i.e., if $\phi > \frac{p - w + w\phi}{p}$, retailer’s probability of achieving its PT will increase. Thus, the retailers prefer to use RS contract to increase their probability of achieving a PT when $\phi > \frac{p - w + w\phi}{p}$. Moreover, it can be seen from Fig. 1 that the condition where RS coefficient is larger, the retailer’s probability of obtaining PT increases.

Fig. 2 The probability of achieving the PT: different PT vs. no contract
3.2 Analysis of the supplier’s decision

Under RS contract the total profit condition of SC upstream supplier can be expressed as follow:

\[
\prod_S = \begin{cases} 
(1 - \varphi)[px + v(q - x)] + w_\varphi q - cq & x < q \\
(1 - \varphi)pq + w_\varphi q - cq & x \geq q 
\end{cases}
\]  \(7\)

Let \(t_s\) denote the expected PT of the supplier. For further analysis of the supplier’s profit function, we know: when \(t_s > \left[ (1 - \varphi)p + w_\varphi - c \right]q\) equivalently \(q < \frac{t_s}{(1 - \varphi)p + w_\varphi - c}\) where the probability is equal \(P(t_s) = 0\); and when \(t_s = \left[ (1 - \varphi)p + w_\varphi - c \right]q\) which is equivalent to \(q = \frac{t_s}{(1 - \varphi)p + w_\varphi - c}\), we have the probability of \(P(t_s) = 1 - F(q)\); where \(P(t_s) = 1 - F\left( \frac{(c - w_\varphi)q - (1 - \varphi)q + t_s}{(1 - \varphi)(p - v)} \right)\), which indicates \(q > \frac{t_s}{(1 - \varphi)p + w_\varphi - c}\), where we have \(P(t_s) = 1 - F\left( \frac{(c - w_\varphi)q - (1 - \varphi)q + t_s}{(1 - \varphi)(p - v)} \right)\). Thus, we have the supplier’s PT probability:

\[
P(t_s) = \begin{cases} 
0 & q < \frac{t_s}{(1 - \varphi)p + w_\varphi - c} \\
1 - F\left( \frac{(c - w_\varphi)q - (1 - \varphi)q + t_s}{(1 - \varphi)(p - v)} \right) & q \geq \frac{t_s}{(1 - \varphi)p + w_\varphi - c}
\end{cases}
\]

**Theorem 2:** Under the RS contract, the optimal order quantity of the upstream supplier is \(t_s\left(\frac{1}{1 - \varphi}p + w_\varphi - c\right)\), and the probability of achieving the PT is \(1 - F\left( \frac{t_s}{(1 - \varphi)p + w_\varphi - c} \right)\).

**Proof of Theorem 2:** For a given order quantity, where \(q\) is \(q < \frac{t_s}{(1 - \varphi)p + w_\varphi - c}\) the supplier’s probability of achieving the PT is equal to 0. When \(q \geq \frac{t_s}{(1 - \varphi)p + w_\varphi - c}\) the inequation \((c - w_\varphi - (1 - \varphi)v) > 0\) always holds. Therefore, \(1 - F\left( \frac{(c - w_\varphi)q - (1 - \varphi)q + t_s}{(1 - \varphi)(p - v)} \right)\) decreases with the increase of the order quantity \(q\). And \(F(x)\) is an increasing function. Thus, the SC supplier’s optimal order quantity is \(t_s\left(\frac{1}{1 - \varphi}p + w_\varphi - c\right)\) and the probability of achieving the PT is \(1 - F\left( \frac{t_s}{(1 - \varphi)p + w_\varphi - c} \right)\).

**Corollary 2:** With the increase of RS coefficient \(\varphi\) supplier’s probability of achieving PT decreases.

**Proof:** By taking the derivative of \(1 - F\left( \frac{t_s}{(1 - \varphi)p + w_\varphi - c} \right)\) with respect to \(\varphi\), we have \(\frac{p(t_s)}{(c - w_\varphi + p(-1 + \varphi))} < 0\) for any \(\varphi\). Thus, the probability of achieving the PT \(1 - F\left( \frac{t_s}{(1 - \varphi)p + w_\varphi - c} \right)\) decreases with the increase of the RS contract coefficient \(\varphi\).

![Fig. 3 SC participant’s probability of achieving the PT under RS coefficient](image-url)
Fig. 3 illustrates a comparison of the SC participants’ PT probabilities with the RS coefficient changes. As can be seen from the Fig. 3 above, the probability of achieving the PT of the retailer increases with the increase of the RS coefficient, while the probability of achieving the PT of the supplier decreases. Moreover, the increasing probability point where the SC participants have the same PT decreases the actual PT. Therefore, an appropriate and coordinated PT will help to achieve a win-win situation.

3.3 Coordination conditions under the revenue-sharing contract

In this subsection, we discussed the SC coordination conditions under the RS contract. To achieve overall SC coordination, the optimal order quantity of the SC participants should be coherent with their PT, so that the optimal order quantity based on the probability of achieving the PT. In this case, under the RS contract conditions, the SC participants can reach their optimal order quantity at the same time they both get the highest probability of achieving their PT. Considering the coordination conditions of the RS contract, we have the following Theorem 3.

Theorem 3: Under the RS contract with SC participants’ PT, the SC coordination condition is \( t_r = -\frac{t_5(p\varphi - w_\varphi)}{c-p+pp\varphi-w_\varphi} \) where \( 0 < (p\varphi - w_\varphi) < p - c \). If the condition falls to \( 0 < (p\varphi - w_\varphi) < \frac{p-c}{2} \), the SC retailer can achieve the higher PT than the upstream supplier; otherwise, i.e., \( \frac{p-c}{2} < (p\varphi - w_\varphi) < p - c \), the PT of retailer is less than that of the SC upstream supplier.

Proof of Theorem 3: Under the conditions where the SC is coordinated, the optimal order quantity that the SC upstream supplier and downstream retailer can achieve the PT are equal \( \frac{t_r}{\varphi p - w_\varphi} = \frac{t_5 (p\varphi - w_\varphi)}{(1-\varphi)p+w_\varphi-c} \). Thus, we obtain \( t_r = -\frac{t_5 (p\varphi - w_\varphi)}{c-p+pp\varphi-w_\varphi} \). For \( t_r > 0, t_5 > 0 \), we have \( \frac{(p\varphi - w_\varphi)}{p-c(p\varphi - w_\varphi)} > 0 \), which is \( 0 < (p\varphi - w_\varphi) < p - c \) or \( p - c < (p\varphi - w_\varphi) < 0 \). However, \( p - c < 0 \) it does not hold; thus, we have \( 0 < (p\varphi - w_\varphi) < p - c \). When \( 0 < (p\varphi - w_\varphi) < \frac{p-c}{2} \) we have \( t_r > t_5 \); when \( \frac{p-c}{2} < (p\varphi - w_\varphi) < p - c \), therefore \( t_r < t_5 \).

According to Section 2 basic model analysis with the three aspects of the problem, this section analyzed the improved conditions of PT oriented SC under the RS contract.

Fig. 4 illustrates SC participants’ probability of achieving PT under different conditions. The horizontal axes represent the PT and the order quantity, respectively. The vertical axes represent the probability of achieving PT. \( P_1 \), \( P_2 \) indicate the maximum probability that the retailer will achieve his PT when no contract condition and under RS contract, respectively; \( P_3 \) indicates the maximum probability that the supplier will achieve her PT under the RS contract condition; \( P_4 \), \( P_5 \) indicate the probability of retailer’s achieving his PT under the different order quantity, when no contract condition and the RS contract condition, respectively; \( q_1, q_2 \) indicate the retailer’s optimal order quantity to achieve a specific PT, under the no-contract condition and the RS contract condition, respectively; \( P_{r0}, P_{r1} \) indicate the retailer’s probability of achieving a specific PT under the no-contract condition and the RS contract condition, respectively; \( \lambda_{s0}, \lambda_{s1} \) indicate the range of PT that the supplier can achieve under the no-contract condition and the RS contract condition, respectively; \( \lambda_{r0}, \lambda_{r1} \) indicate the range of PT that the SC retailer can achieve under the no-contract condition and the RS contract condition, respectively.

From the Fig. 4, the following conclusions can be seen:

- For the SC retailer, under the RS contract the probability of achieving a specific PT is higher, \( P_{r1} > P_{r0} \); the range of PT that the retailer can achieve under the RS contract will also increase \( \lambda_{r1} > \lambda_{r0} \);
- For the supplier, under the no-contract condition the probability of achieving specific PT is 100%, under RS contract condition the range of PT that the supplier can achieve will also increase \( \lambda_{s1} > \lambda_{s0} \);
- Under the RS contract condition the SC retailer’s optimal order quantity \( q_2 \) is less than the optimal order quantity \( q_1 \) when non-contractual condition. However, the SC retailer’s probability of achieving his PT will increase, because of the supplier’s RS portion.
Fig. 4 Analysis of PT oriented SC coordination conditions under the RS contract.

4. Buyback contract with a specific profit target

In this section, we discussed the buyback contract under the SC participants’ PT. Firstly, we constructed the buyback contract conditions for the SC downstream retailer and upstream supplier respectively. Then we analyzed the coordination conditions under the buyback contract within SC participants’ PT.

Based on the Definition 1, the events of the decision-making process under the buyback contract with SC participants’ PT and based on the Definition 2, the analyzes of the coordination conditions under the buyback contract with SC participants’ PT are as follows:

1) SC downstream retailer and upstream supplier set their PTs;
2) The upstream supplier shares the market risk and will buy back the unsold products.
3) SC retailer and supplier will determine their PT and set the optimal order quantity.
4) According to Definition 2, obtaining the PT based RS contract coordination conditions.
5) Analyzing the RS contract conditions for the improvement of the overall SC profitability.

4.1 Analysis of the retailer’s decision

Under the buyback contract, the supplier charges a retailer wholesale price \( w_b \) for each unit ordered product and provides the buyback credit \( b \) for each unit remaining product at the end of a selling season. Thus, the profit conditions of the SC downstream retailer can be expressed as follow:

\[
\prod_r = \begin{cases} 
px + (b + v)(q - x) - wbq & x < q \\
pq - wbq & x \geq q 
\end{cases} \tag{8}
\]

Considering the contract conditions within PT probability we can get the followings. When the retailer’s PT is \( t_r > (p - wb)q \) which is \( q < \frac{tr}{p-wb} \), the retailer’s PT probability is equal to \( P(t_r) = 0 \)

When PT is \( t_r = (p - wb)q \), which is \( q = \frac{tr}{p-wb} \), we have the retailer’s PT probability \( P(t_r) = 1 - F(q) \) when \( t_r \leq (p - w)q \) i.e. \( q > \frac{tr}{\varphi p-wb} \) we have \( P(t_r) = 1 - F \left( \frac{(w_b-b-v)q+tr}{p-b-v} \right) \). Thus, we have overall probability condition of retailer’s PT as follow: \( P(t_r) = \begin{cases} 
0 & q < \frac{tr}{p-wb} \\
1 - F \left( \frac{(w_b-b-v)q+tr}{p-b-v} \right) & q \geq \frac{tr}{p-wb} 
\end{cases} \)

**Theorem 4:** Under the buyback contract, the SC retailer’s optimal order quantity with his PT is \( t_r \frac{tr}{p-wb} \), and the probability of achieving the PT is \( 1 - F \left( \frac{tr}{p-wb} \right) \).

**Proof of Theorem 4:** In the case where order quantity \( q < \frac{tr}{p-wb} \), retailer’s probability of achieving the PT is equal to 0. In the next case where order quantity \( q \geq \frac{tr}{p-wb} \), the probability \( \frac{w_b-b-v}{p-b-v} > 0 \).
Thus, \( \frac{(w_b - b - v)q + t_r}{p - b - v} \) increases with the increase of \( q \). And \( 1 - F \left( \frac{(w_b - b - v)q + t_r}{p - b - v} \right) \) decreases with increase of order quantity \( q \). \( F(x) \) is an increasing function. Therefore, retailer’s optimal order quantity is \( \frac{t_r}{p - w_b} \) and the probability of achieving PT is \( 1 - F \left( \frac{t_r}{p - w_b} \right) \).

**Corollary 3:** The SC retailer’s probability of achieving his PT cannot increase under the buyback contract conditions.

**Proof:** In case of no contract, the supplier charges downstream retailer with the wholesale price \( w \) for unit product. While under the buyback contract, supplier charges downstream retailer the wholesale price \( w_b \) for unit product and provides buyback credit \( b \) for remaining product at the end of selling season. Compared to the case where the contract is not applied, the supplier will generate a transfer payment to the downstream retailer at the end of selling season due to the buyback contract conditions. Therefore, to ensure the profit, the upstream supplier must charge higher wholesale price, i.e., \( w_b > w \). Similarly, under the RS contract, the upstream supplier will receive a portion of the retailer’s revenue for each unit sold, so the supplier will be willing to sell at a lower wholesale price, i.e., \( w_\phi < w \). Consequently, we have \( w_\phi < w < w_b \).

In order to increase the retailer’s PT probability, the contract condition has to satisfy \( 1 - F \left( \frac{t_r}{p - w_b} \right) > 1 - F \left( \frac{t_r}{p - w} \right) \), that is \( \frac{t_r}{p - w_b} > \frac{t_r}{p - w} \). Thus, we have \( w_b < w \) which is a contradiction with the premise \( w_\phi < w < w_b \). Hence, the probability that the retailer achieves his PT cannot increase using a buyback contract. Therefore, compared with the situation under the RS and buyback contracts, the SC downstream retailer prefers to choose the RS contract rather than the buyback contract.

### 4.2 Analysis of the supplier’s decision

Under the buyback contract the total profit of the SC upstream supplier within the newsboy problem can be expressed as follow:

\[
\prod_{s} = \begin{cases} 
-b(q - x) + w_bpq - cq & x < q \\
wbq - cq & x \geq q 
\end{cases}
\]  

(9)

With the supplier’s PT, when \( t_s > (w_b - c)q \), i.e., \( q < \frac{t_s}{(w_b - c)} \), we have \( P(t_s) = 0 \); and when \( t_s = (w_b - c)q \), i.e., \( q = \frac{t_s}{(w_b - c)} \), we have \( P(t_s) = 1 - F(q) \); finally, when the supplier’s PT is \( t_s \leq (w_b - c)q \), i.e., \( q > \frac{t_s}{(w_b - c)} \) we have \( P(t_s) = 1 - F \left( \frac{(w_b - b - c)q + t_s}{b} \right) \). Thus, we have the supplier’s PT probability: \( P(t_s) = \begin{cases} 
0 & q < \frac{t_s}{(w_b - c)} \\
1 - F \left( \frac{(w_b - b - c)q + t_s}{b} \right) & q \geq \frac{t_s}{(w_b - c)} 
\end{cases} \).

**Theorem 5:** Under the buyback contract, the optimal order quantity for the supplier is \( q = \frac{t_s}{(w_b - c)} \) and the SC supplier’s PT probability is \( P(t_s) = 1 - F \left( \frac{t_s}{(w_b - c)} \right) \).

**Proof of Theorem 5:** For given order quantity \( q \), when \( q < \frac{t_s}{(w_b - c)} \) SC upstream supplier’s probability of PT is equal to 0. When \( q \geq \frac{t_s}{(w_b - c)} \), the inequation \( (w_b - b - c) > 0 \) always holds. Therefore, \( \frac{(w_b - b - c)q + t_s}{b} \) increases with the increase of order quantity. \( 1 - F \left( \frac{(w_b - b - c)q + t_s}{b} \right) \) decreases with the increase of order quantity \( q \). \( F(x) \) is an increasing function. Thus, the supplier’s optimal order quantity is \( \frac{t_s}{(w_b - c)} \) and the probability of achieving PT is \( 1 - F \left( \frac{t_s}{w_b - c} \right) \).

Compared to the case under RS contract where the supplier’s optimal order quantity is \( \frac{t_s}{(1 - \phi)p + w_\phi - c} \) the probability of achieving PT is \( 1 - F \left( \frac{t_s}{(1 - \phi)p + w_\phi - c} \right) \).
When \( w_b > (1 - \varphi)p + w_\varphi \), the supplier’s probability of PT under the buyback contract is greater than under the usage of a RS contract, otherwise, i.e., \( w_b < (1 - \varphi)p + w_\varphi \), the supplier’s probability of PT under the buyback contract is less than the RS contract condition.

4.3 Coordination condition under the buyback contract

PT oriented SC coordination conditions under the buyback contract are the same as the RS contract condition, where the optimal order quantity of the SC participants should be coherent with their predetermined PT. In this case, the SC participants can reach their optimal order quantity and achieve a higher probability of their PT.

**Theorem 6:** Under the buyback contract parameters within the PT, the SC coordination condition is \( t_r = \frac{t_s(p-w_b)}{(w_b-c)} \). When \( w_b > \frac{p+c}{2} \) the SC downstream retailer’s achieving PT is higher than the upstream supplier; otherwise, i.e., \( w_b < \frac{p+c}{2} \), the downstream retailer’s achieving PT is smaller.

**Proof of Theorem 6:** When SC is coordinated under the buyback contract, the optimal order quantity that the SC participants can achieve the PT is equal, i.e., \( t_r = \frac{t_s(p-w_b)}{(w_b-c)} \). Thus, the coordination condition is \( t_r = \frac{t_s(p-w_b)}{(w_b-c)} \). When \( w_b > \frac{p+c}{2} \), the SC retailer can achieve a higher PT than the upstream supplier, i.e., \( t_r > t_s \); When \( w_b < \frac{p+c}{2} \), the PT of the downstream retailer is less than that of the SC upstream supplier, i.e., \( t_r < t_s \).

5. Numerical analysis

In this section, we numerically analyzed the contract selection with the given PT for both SC participants. We assumed that \( c = 30 \), \( p = 50 \), \( \varphi = 15 \) and market demand \( x \) is uniformly distributed with \( U(0,200) \). The density function and the cumulative distribution function of the stochastic demand \( x \) are respectively \( f(x) = \frac{1}{200} \) and \( F(x) = \frac{x}{200} \). We assumed that the SC downstream retailer’s PT is \( t_r = 800 \) and the upstream supplier’s PT is \( t_s = 1000 \). Following the Theorem 3, we assumed with the difference of \( t_r = 800 < t_s = 1000 \), where \( \frac{p-c}{2} < (p\varphi - w_\varphi) < p - c \), and where we have \( 30 < w_\varphi < 40 \).

For the RS contract analysis, the SC downstream retailer’s optimal order quantity is assumed as \( \frac{800}{50\varphi - w_\varphi} \) following to Theorem 1. Where the probability of achieving the PT of 800 is \( 1 - \frac{4}{50\varphi - w_\varphi} \).

For the SC upstream supplier, following to Theorem 2 the optimal order quantity is \( \frac{5}{50(1-\varphi) + w_\varphi - 30} \) and the probability of achieving the PT of 1000 is \( 1 - \frac{5}{50(1-\varphi) + w_\varphi - 30} \). Thus, we can analyze the RS condition \( \varphi = \frac{804+9w_\varphi}{450} \).

In Table 1, the first-row numerical values show the condition where the RS contract can coordinate the PT based SC. In this case, the probability that the SC retailer and supplier can achieve their PT are equal. With the same wholesale price but under the different RS coefficient the probability of the SC retailer’s PT is increasing with the increase of the RS coefficient. The probability of supplier’s PT decreases with the increase of the RS coefficient and increases with the increase of the wholesale price. This table shows that retailers prefer the contract parameters combination with high revenue sharing coefficient and low wholesale price, while suppliers prefer the opposite.

For the buyback contract analysis, the SC downstream retailer’s optimal order quantity is assumed as \( \frac{800}{50-w_b} \) following to Theorem 4. Where the probability of achieving PT of 800 is \( 1 - \frac{4}{50-w_b} \).

For the SC upstream supplier, following to Theorem 5 the optimal order quantity is \( \frac{1000}{w_b-30} \) and the probability of achieving the PT of 1000 is \( 1 - \frac{5}{(w_b-30)} \). Then, according to Theorem 6, we can analyze the coordination condition under buyback contract as \( w_b = \frac{370}{9} \).
Table 1 SC participants PT probabilities within the RS contract parameters

<table>
<thead>
<tr>
<th>$w_p$</th>
<th>$q$</th>
<th>$q^*$</th>
<th>$P(t_r)$</th>
<th>$P(t_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.00</td>
<td>0.88</td>
<td>90.00</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>35.00</td>
<td>0.89</td>
<td>84.21</td>
<td>0.58</td>
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<td>80.00</td>
<td>0.60</td>
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<td>0.47</td>
</tr>
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<td>0.64</td>
<td>0.44</td>
</tr>
<tr>
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<td>0.65</td>
<td>0.41</td>
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<tr>
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<td>0.94</td>
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<td>0.67</td>
<td>0.38</td>
</tr>
<tr>
<td>35.00</td>
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<td>64.00</td>
<td>0.68</td>
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<tr>
<td>35.00</td>
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<td>0.69</td>
<td>0.29</td>
</tr>
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<tr>
<td>39.00</td>
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<td>160.00</td>
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<td>0.67</td>
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</table>

Table 2 SC participants PT probabilities within the buyback contract parameters

<table>
<thead>
<tr>
<th>$w_p$</th>
<th>$q^*$</th>
<th>$P(t_r)$</th>
<th>$P(t_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.11</td>
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<td>0.55</td>
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<tr>
<td>40.50</td>
<td>84.21</td>
<td>0.58</td>
<td>0.52</td>
</tr>
<tr>
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<td>80.00</td>
<td>0.60</td>
<td>0.50</td>
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<tr>
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<td>76.19</td>
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<tr>
<td>39.00</td>
<td>72.73</td>
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<td>38.50</td>
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<tr>
<td>38.00</td>
<td>66.67</td>
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<tr>
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<td>37.00</td>
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<td>0.71</td>
<td>0.17</td>
</tr>
</tbody>
</table>

In Table 2, the first-row numerical values show the condition where the buyback contract can achieve coordination. At this point, the probabilities that the SC participants can achieve their PT are equal. Table 2 shows that under the buyback contract parameter $w_p$, the probability of SC downstream retailer’s PT increases with the increase of the wholesale price, while the probability of the SC upstream supplier PT decreases. This table shows that retailers prefer contract parameters with high wholesale prices, while suppliers prefer the opposite.

6. Conclusion

The traditional SC research is based on the expected utility theory, where the expected utility maximization as the decision-making goal. Most of the studies are based on the statistical average value and thus cannot avoid the low returns or large fluctuations in profitability. For decision-makers, maximizing the probability of achieving the PT can effectively reduce their own risk. However, from the perspective of SC, each decision-maker has its own goal. There is a certain conflict between the goal of decentralized decision-making and the overall optimization of the SC. Therefore, this paper proposes the research of SC contracts based on PT.

In this study, we investigated the RS and buyback contracts based on the SC participants’ PT. Different from the traditional SC contracts, it has valuable advantages for SC participants to deal with market risk. Through the research of SC contractual coordination under the PT, our main findings can be summarized as follows:
• This study analyzed the RS contract and buyback contract with PT and obtained the optimal order quantity and the probability of achieving PT for both SC participants. Additionally, we analyzed the coordination conditions of the RS contract and buyback contract with the PT.

• When the RS coefficient of the contract is within a certain range, the probability of achieving the PT can be increased for both SC participants. As the RS coefficient increases, the probability of SC retailer’s PT increases, and the upstream supplier’s probability of PT decreases.

• From the comparison, it can be seen that under the buyback contract a SC retailer cannot increase the probability of achieving the PT by adjusting the contract parameters. Therefore, under the PT strategy, the retailer is more willing to adopt the RS contract rather than the buyback contract.

• Under the buyback contract, the SC supplier can adjust the contract parameters within a certain range to increase the probability of achieving her PT. Compared with the RS contract, the SC upstream supplier’s contract selection decision depends on the specific contract parameters.

• Coordination conditions analyses under the two main contracts proved that the PT of the SC participants can be achieved based on the contracts’ parameters. This study found the unique condition where both SC participants can achieve their PT and coordinate overall SC.

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References


